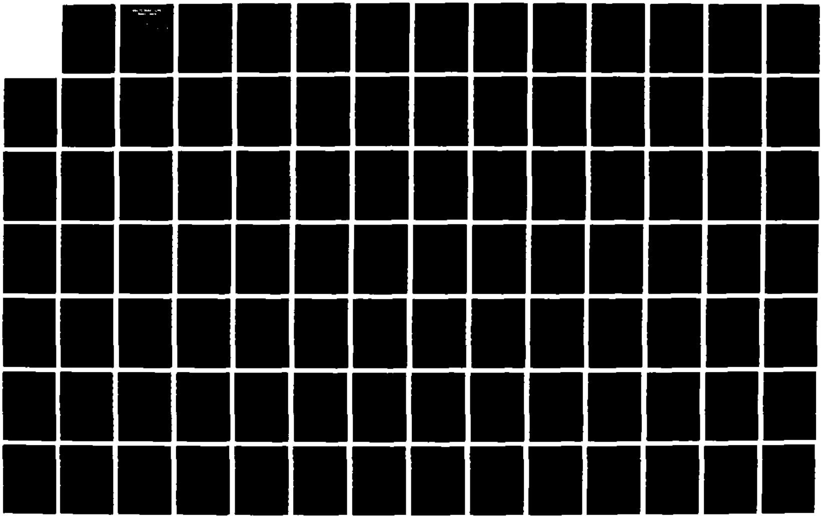
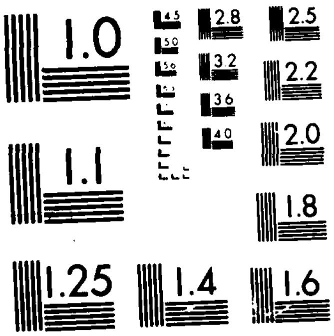
AD-A193 526 ANALYSIS OF THE EFFECT OF FAULTY SPARES ON THE PERFORMANCE OF DIAGNOSTIC ALGORITHMS IN RELIABLE SYSTEMS(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA 1/2

UNCLASSIFIED M PAKTUNA DEC 87 F/G 12/6 NL




1.0
4.5
5.0
5.6
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

ANALYSIS OF THE EFFECT
OF FAULTY SPARES ON THE PERFORMANCE OF
DIAGNOSTIC ALGORITHMS IN RELIABLE SYSTEMS

by

Mustafa Paktuna

December 1987

Thesis Advisor: Jon T. Butler

Approved for public release; distribution is unlimited

Analysis of the Effect of Faulty Spares on
The Performance
of Diagnostic Algorithms in Reliable Systems

by

Mustafa Paktuna
Lieutenant Junior Grade, Turkish Navy
B.S., Turkish Naval Academy, 1981

Submitted in partial fulfillme... of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1987

Author: M Paktuna
Mustafa Paktuna

Approved by: Jon T. Butler
Jon T. Butler, Thesis Advisor

Mitchell L. Cotton
Mitchell L.Cotton, Second Reader

John P. Powers, Chairman,
Department of Electrical
and Computer Engineering

G E Schacher
Gordon E. Schacher
Dean of Science and Engineering

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b RESTRICTIVE MARKINGS

2a SECURITY CLASSIFICATION AUTHORITY

2b DECLASSIFICATION / DOWNGRADING SCHEDULE

3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.

4 PERFORMING ORGANIZATION REPORT NUMBER(S)

5 MONITORING ORGANIZATION REPORT NUMBER(S)

6a NAME OF PERFORMING ORGANIZATION
Naval Postgraduate School

6b OFFICE SYMBOL (If applicable)
62

7a NAME OF MONITORING ORGANIZATION
Naval Postgraduate School

6c ADDRESS (City, State, and ZIP Code)
Monterey, California 93943-5000

7b ADDRESS (City, State, and ZIP Code)
Monterey, California 93943-5000

8a NAME OF FUNDING / SPONSORING ORGANIZATION

8b OFFICE SYMBOL (If applicable)

9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c ADDRESS (City, State, and ZIP Code)

10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)
ANALYSIS OF THE EFFECT OF FAULTY SPARES ON THE PERFORMANCE OF DIAGNOSTIC ALGORITHMS IN RELIABLE SYSTEMS

12 PERSONAL AUTHOR(S)
Mustafa Paktuna

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Master's Thesis	FROM ______ TO ______	1987 December	140

16 SUPPLEMENTARY NOTATION

17 COSATI CODES

FIELD	GROUP	SUB-GROUP

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Diagnostic Algorithm; Asymptotic Approximation

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Previous research of systems diagnosis algorithms have assumed that replacement processors are fault-free. In practical applications, however, faults can occur in spare processors. It is shown that faulty spare processors have a surprisingly large deleterious effect on the speed of diagnosis in the universal diagnosis Algorithm-1 analyzed by Smith [Ref.13].

In this thesis we derive asymptotic approximation to the probability of repair when faulty spares are present. An exact value can be obtained from previously known results. Our asymptotic approximations yield good estimates that can be calculated quickly. The analysis was performed by formulating the probability of repair calculations as a multiplication of matrices and by deriving approximations to the largest eigenvalues of these matrices. Also, faster calculations were achieved by an aggregation operation on the states of the system.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21 ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a NAME OF RESPONSIBLE INDIVIDUAL	22b TELEPHONE (Include Area Code)	22c OFFICE SYMBOL
Prof. Jon T. Butler	(408)625-2431	62Bu

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

ABSTRACT

Previous research of systems diagnosis algorithms have assumed that replacement processors are fault-free. In practical applications, however, faults can occur in spare processors. It is shown that faulty spare processors have a surprisingly large deleterious effect on the speed of diagnosis in the universal diagnosis Algorithm_1 analyzed by Smith [Ref.13]. Algorithm_1 is described as follows; "Replace a processor if it fails at least one test." The speed of diagnosis is nearly independent of the distribution of fault processors. That is, as long as the total number of fault processors is constant, the probability of repair is relatively unaffected by whether more faulty processors are in the spares or in the system.

In this thesis we derive an asymptotic approximation to the probability of repair when faulty spares are present. An exact value can be obtained from previously known results. However, the calculations are extremely time consuming with a time complexity of order $O(4^n)$, where n is the number of processors. Our asymptotic approximations yield good estimates that can be calculated quickly. The analysis was performed by formulating the probability of repair calculations as a multiplication of matrices and by deriving approximations to the largest eigenvalues of these

matrices. Also, faster calculations were achieved by an aggregation operation on the states of the system.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

GLOSSARY

o	Fault-free node
*	Faulty node
k	The number of steps to repair the system
n	The number of nodes in the system
nf	The number of faulty nodes in the system
m	The number of spare nodes in the system
mf	The number of faulty spare nodes in the system
s	The number of states to repair the system
P_s	The probability of being in state s before diagnosis
P_s'	The probability of being in state s after diagnosis
t	The number of test links for each node
P	Probability matrix of transition states
Pa	Probability matrix of aggregated transition states
P^k	The probability of transition states after k applications of the diagnostic algorithm
V	Eigenvector
E	Eigenvalue
D	The matrix of eigenvalues of the transition matrix P

C The matrix of eigenvectors of the transition matrix P

C^{-1} Inverse matrix of eigenvectors of the transition matrix P

ACKNOWLEDGEMENTS

It is with great respect and admiration that I wish to express my sincere thanks to Professor Jon T. Butler for his guidance, assistance, understanding and encouragement in the development of this thesis. His many valuable suggestions made during the course of this research are gratefully acknowledged.

I would like also to thank The Turkish Naval Academy for giving me the opportunity to study at the Naval Postgraduate School. The support of my friend, Brenda Orr, who helped with English construction, is highly appreciated.

Finally, I would like to thank my wonderful parents to whom therefore this work is dedicated.

I. INTRODUCTION

A. GENERAL

In the past decade the concept of fault tolerance in digital systems has received considerable attention. This interest has been motivated by the need for computers that operate properly under hardware failures and software errors. Reliability is a measure of the probability that a system will operate without failure. Several methods have been used in order to achieve reliability. One method is through redundancy where identical units perform identical computations and the results are compared. Redundancy is quite expensive and hence is suitable primarily for the situation when repair is difficult or even impossible, i.e., space missions. Another method is to use fault diagnosis and repair. In this approach the fault must first be detected and then the faulty component must be identified. However, for real time critical applications in which fast fault-free operation is necessary, the repair must be very fast and possibly not involve human intervention, i.e., automatic self-repair. In order to achieve automatic repair, either we can use spare components or we can design into the system a capability of reconfiguration in order to work with reduced capacity in the presence of the faults. Self-repairing implies self-diagnosability, i.e., the

capability of automatic detection and diagnosis of faults to a number of replaceable nodes. Detection and isolation of faults involves applying tests where a test consists of applying a set of inputs to the system, observing the resulting outputs, and comparing the outputs with their anticipated values.

The development of VLSI technology makes it possible to partition a system into replaceable nodes and the advent of low cost microprocessors makes possible networks of hundreds (or more) of interconnected nodes. The demand for high availability and self-diagnosability is becoming a feature of major importance in digital systems. Each node can be tested by some combination of other nodes. There have been several attempts to formulate repair strategies for digital system fault diagnosis.

B. INTRODUCING THE PROBLEM

Previously studies of systems diagnosis algorithms have assumed that replacement processors are fault-free. However, in practice faults can exist in spare processors. Lee and Butler [Ref. 19] have shown that faulty spare processors have significant negative effect on the speed of diagnosis in the Algorithm_1 analyzed in Smith [Ref. 13]. In this work, we develop a procedure for formulizing an optimal design with respect to speed of diagnosis in a system which consists of replaceable nodes, and we consider

the number of steps for repairing a system. Our analysis also shows asymptotic approximations to the probability of repair which require eigenvalues produced by the matrix manipulation program MATLAB. However, there is a limit on the dimension of matrices which MATLAB can handle. An n node system requires a matrix of dimension $2^n \times 2^n$, which is very large for even moderate size n, as shown in Table 1.

TABLE 1

OF NODES IN THE SYSTEM VS
TRANSITION MATRIX DIMENSION

# OF NODES	TRANSITION MATRIX DIMENSION
2	4 X 4
3	8 x 8
4	16 x 16
5	32 x 32
6	64 x 64
.	. .
.	. .
.	. .
10	1024 x 1024
.	. .
15	32,768 x 32,768
.	. .
20	1,048,576 x 1,048,576
.	. .
.	. .
.	. .
.	. .
n	billions x billions

C. CONTRIBUTION AND ORGANIZATION OF THE THESIS

Many diagnostic models measure diagnosability by the number of faults a system can tolerate in the worst case. Additionally, a very conservative philosophy of system repair is assumed, i.e., only faulty nodes can be replaced. An alternative repair strategy [Ref. 13] involves inexact diagnosis and replacement of faulty nodes plus some nodes which may not be faulty.

The goal of this thesis is to derive a probability of repair calculation that can be performed in reasonable time. Previously, exact calculations were available, but required extensive computer time and memory. Our focus is on Algorithm_1 of Smith [Ref. 13]. In Chapter II, we survey the literature on system diagnosability. In Chapter III, we present a sequential diagnosis of single loop systems when Algorithm_1 is applied. In Chapter IV, we derive the transition matrix of the probability of repair using Algorithm_1. In Chapter V, we present the formulation of our first approximation. In Chapter VI, we introduce the aggregation of the transition matrix P. Conclusions and some suggestions for future work are presented in Chapter VII.

II. BACKGROUND AND NOTATION

A. SURVEY OF THE LITERATURE ON SYSTEM DIAGNOSABILITY

In the systems diagnosis model of reliable multiprocessing systems, faulty processors are identified from test results produced by other processors in the system [Ref. 1]. The goal of the diagnosis algorithm is to replace possibly faulty processors with spares so that the system consists entirely of fault-free processors.

In all previous systems diagnosis models, it is assumed that, when a processor is replaced, its replacement is fault-free. While this assumption simplifies the analysis, it is not realistic. In a practical system, the most reliable components will be used to do the computation, while the least reliable ones are kept as spare processors. We consider the effect faulty spares have on the speed of diagnosis. When there are even few faulty spares, a significant degragadation in speed of diagnosis can occur.

A SYSTEM is a directed graph in which nodes represent processors and arcs represents tests between processors. Let $\{U_0, U_1, \ldots U_{n-1}\}$ be the set of n processors. Associated with each node is a status, faulty (bad) and fault-free (good). If there is an arc from U_i to U_j, then U_i tests U_j. Associated with each arc is a test outcome, which is generated as follows:

The outcomes of the tests are represented by binary values on the arcs (a_{ij}) where a_{ij} is '0' if U_i evaluates U_j to be fault-free, and is '1' if U_i evaluates U_j to be faulty. If U_i itself is faulty then the evaluation of U_j is unreliable, since a_{ij} can assume '0' or '1' regardless of the status of U_j. If U_i is fault-free, its test results are correct; it passes fault-free processors it tests and fails faulty processors.

$a_{ij} = 1$ if U_i evaluates U_j as faulty,

and

$a_{ij} = 0$ if U_i evaluates U_j as fault-free.

Figure 2.1 summarizes these assumptions. Here a open circle represents a fault-free processor and an asterisk represents a faulty processor. For example, the topmost arrow between open circles represents a test by a fault-free processor U_i of a fault-free processor U_j. It follows from the previous discussion that the test outcome a_{ij} is 0. This is shown in Figure 2.1.

Figure 2.2 shows an example of a five processor multiprocessing system where each processor is tested by exactly one other one. It is called a single loop system. In Figure 2.2 the complete set of test outcomes is shown. Such set is called a SYNDROME.

The object of a diagnosis algorithm is to determine which nodes are faulty given the syndrome. Because of the arbitrary test results by faulty nodes, it may be impossible

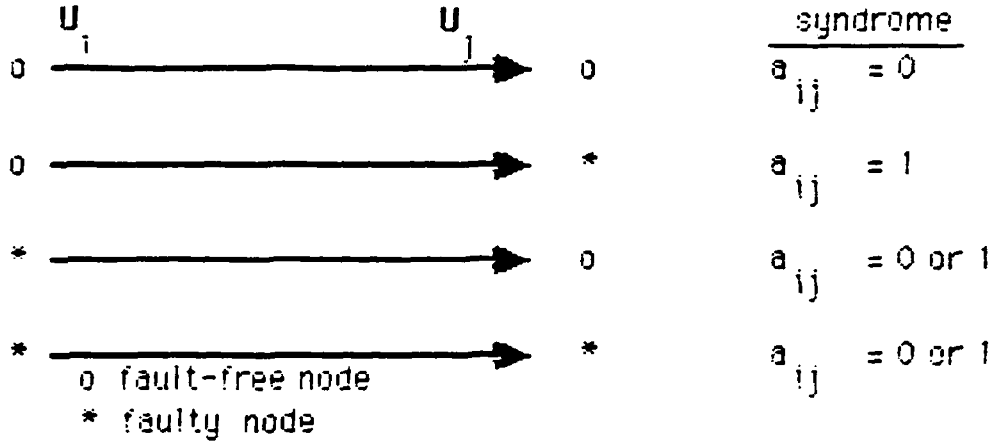


Figure 2.1 Assumed test outcomes in the Preparata-Metze-Chien model [Ref. 1]

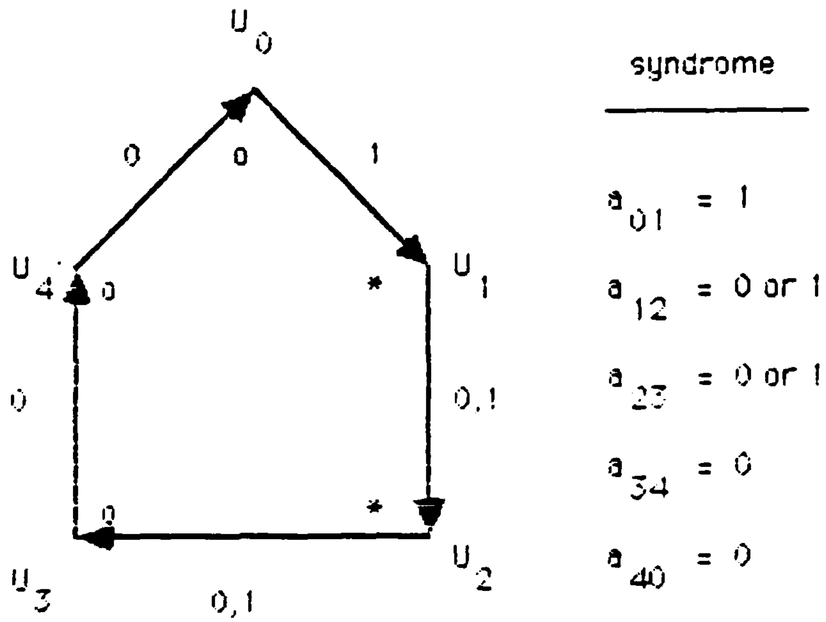


Figure 2.2 A five_processors single loop system and associated test outcomes.

to uniquely identify the faulty nodes. A step in the algorithm consists of identifying nodes to be replaced, replacing them with spare nodes, and generating the next syndrome. The diagnosis is complete when all test results are 0. It is assumed that no fault-free node becomes faulty during diagnosis.

Figure 2.2 shows how the presence of faulty processors inhibits the identification of faulty processors. For example, in Figure ? ? there are three fault-free processors (open circles) and two faulty processors (asterisks). If both faulty processors produce pass (0) test results we have a syndrome that is also produced by the set shown except that U_2 is fault-free. It is not known beforehand exactly which processors are faulty, and there is an ambiguity. We could resolve this by replacing U_1 only and applying tests again. Since after this U_1 is fault-free, we can identify U_2 as faulty and replace it, thus completing the diagnosis. However, this requires two diagnosis steps.

Mallela and Masson [Ref. 9] have considered a system in which faults are intermittent. That is, at the time of one test application a tested node U may be faulty, while at a later time it may be fault-free. Butler [Ref. 16] developed the relationships between diagnosability of general systems. That is, three models are compared; one model is the conventional system with binary-valued test outcomes, the other has three valued test outcomes, (where the third value

is missing test result) and the third model corresponds to permanently and intermittently faulty processors. Also Butler [Ref. 15] extended the technique to the case where intermittently, as well as permanently, faulty processors are presented. Karunanithi and Friedman [Ref. 11] considered replacement algorithms for the Preparata-Metze-Chien model in which fault-free nodes could be replaced. All previous studies assumed that no fault-free node could be replaced. Chwa and Hakimi [Ref. 14] analyzed in detail diagnosis under the conditions proposed by Karunanithi and Friedman [Ref. 12]. In both studies, the goal of the diagnosis strategy is to determine the smallest set of nodes which contain all faulty nodes and perhaps some good nodes as well. Maheshwari and Hakimi [Ref. 9] considered a probabilistic approach for fault diagnosis.

III. SEQUENTIAL DIAGNOSIS

A. k_STEP t/s_FAULT DIAGNOSABLE SYSTEMS

A new measure of system diagnosis, t/s diagnosability originally proposed by Friedman [Ref. 6], is used to study the diagnosability of digital systems. Friedman used this measure to study a canonical class of systems, "Single Loop Systems." Generally, the system is represented and analyzed by means of the graph theoretic model of Preparata et al. [Ref. 1].

Let S be a single loop system, a system which has exactly one test link outgoing from node U_i, $i = 0, 1, 2 \ldots, n-1$, which is connected to node $U_{i+1(\mathrm{mod}\ n)}$ where n is the number of nodes, as in Figure 3.1

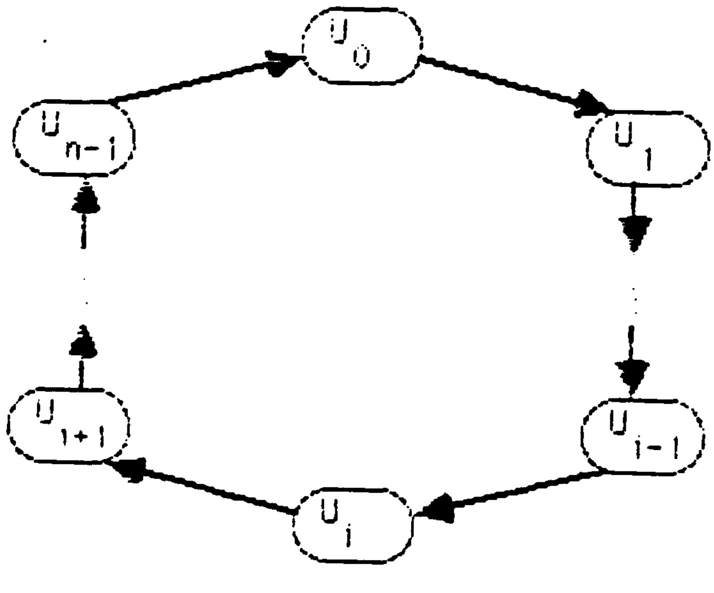


Figure 3.1 A single loop system

The question we wish to resolve is how this system should be repaired. We assume that the probability that i nodes are faulty is less than the probability that i+1 nodes are faulty.

Friedman defined a new measure of system diagnosability related to this concept.

Definition 1: A system S is k_step t/s (t-out-of-s) diagnosable if and only if by no more than k applications of the diagnostic tests any set of nf < t faulty nodes in S can be diagnosed and repaired by replacing at most s nodes.

Obviously, $s \geq t$ and $n \geq s$. If $s = n$, repair is trivial since the entire system is replaced. A measure of the diagnosability of the system is the average or the maximum value of s-nf over all system fault conditions with $nf \leq t$ faulty nodes.

Here we will introduce three replacement algorithms which have been presented in Refs. 1, 12, 13, and 18. Algorithm_1 and 2 are similar in Smith [Ref. 13]. The algorithms are as follows:

Algorithm_1 : At each repair step, perform the tests and replace the nodes which fail <u>at least one test with randomly selected nodes from spares</u>. Replaced nodes are placed back into the set of spares. If all test results are 0 (pass), then the system is assumed to be correct.

Algorithm_2 : At each repair step, perform the tests and replace the nodes which fail <u>the maximum number of tests</u>. Replaced nodes are placed back into the set of spares. If all test results are 0 (pass), then the system is assumed to be correct.

Algorithm_3 : At each repair step, perform the tests and replace the nodes which fail <u>the maximum number of tests. If the number of nodes in Spare-1 is not enough, randomly select any additional needed spare nodes from Spare-2.</u> Replaced nodes are placed back into the Spare-2. If all test results are 0 (pass), then the system is assumed to be correct. Initially, all spare units are in Spare-1, and Spare-2 is empty.

A good sequential diagnosis strategy will identify many faulty nodes for replacement at each diagnosis step. It has been shown [Refs. 13, 15, 18] that Algorithm_1 is significantly faster than Algorithm_2 when spares are fault-free, while the reverse is true when some spares are faulty. Algorithm_3 is faster than either Algorithm_1 and Algorithm_2, in the presence of faulty spares; it was

designed to compensate for the negative effect of faulty spares.

B. SEQUENTIAL DIAGNOSIS OF SINGLE LOOP SYSTEMS USING ALGORITHM_1

In this work, we selected Algorithm_1 because it results in the most tractable analysis. We assume symmetric invalidation [Ref. 1] where the test outcome of a test by a faulty node is 0 or 1. The possibility of each occurrence is assumed to be 1/2.

Repair means the replacement of possibly faulty (bad) nodes by a set of spare processors. For sequential diagnosis, every time that a repair step has been terminated all tests will be performed to check whether the system is correct or not. It is assumed that, for more practical use, even if both nodes U_i and U_j are left untouched during a replacement process, U_i will be tested again at the next step because some permanent faulty nodes could become faulty.

As mentioned before, Algorithm_1 is: "Replace a node if it fails at least one test." A repair step in the algorithm consists of identifying nodes to be replaced, replacing them by spares some of which are faulty. All replaced nodes are placed back into the set of spares since they might contain fault-free nodes. We do not assume that the new replacing node is fault-free. That is, there can initially exist

faulty units in the set of spares. This situation is shown in Figure 3.2. An arrow from the spares to the system represent a choice of spares to use as replacement processors. An arrow from the system to the spares represents the placement of replaced processors back into the set of spares.

Another problem created by the presence of faulty spares is a degradation in speed and efficiency of diagnosis. In t' ≥ worst case, if only faulty units are involved in the replacement, then an infinite cycling of the procedure is possible. The problem can be solved by having sufficiently many fault-free spares.

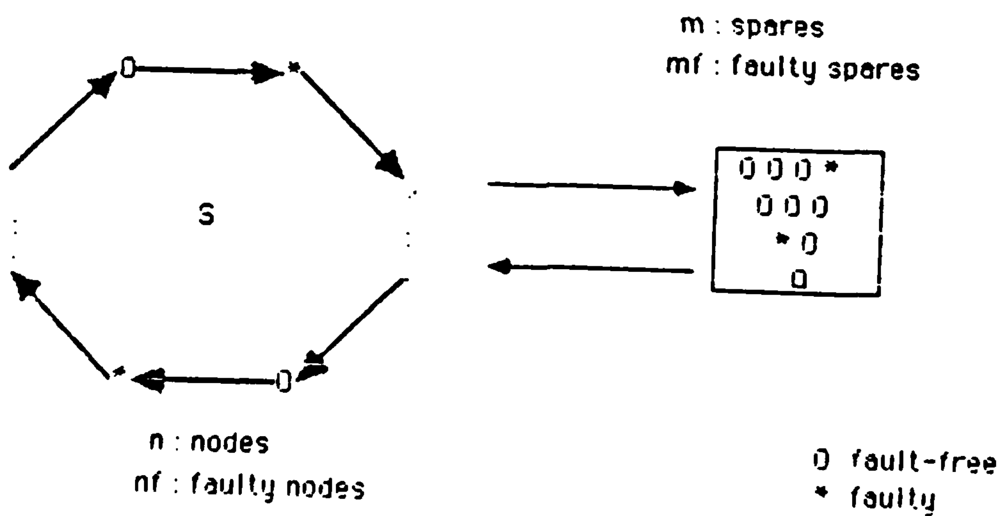


Figure 3.2 Sequential Diagnosis of single loop systems using Algorithm_1.

IV. CALCULATION OF THE PROBABILITY OF REPAIR

A. METHOD OF APPROACH

We seek the probability of producing a completely fault-free system after the application of the Algorithm_1 given that there is a fixed number of initially faulty nodes in the system and a fixed number of faulty spares. Because of faulty spares, on each application of the Algorithm_1, faulty nodes can be introduced into the system. Further, Algorithm_1 may incorrectly assess the status of a node and so fault-free nodes may be replaced by faulty nodes. Therefore, it is possible that an application of the algorithm can produce a net increase in the number of faulty nodes. In calculating the probability of repair [Ref. 18], we assume:

1. Any arrangement of a fixed number of initially faulty nodes is as likely as any other arrangement.

2. Each test by a faulty node is pass or fail with probability 50%. Test results are generated at each step of the Algorithm_1, so that the test result by faulty node U_i of node U_j may not be the same after the application of the Algorithm_1 as before, even though neither U_i nor U_j is replaced. Such behavior is a characteristic of intermittent faults.

3. Spares are chosen randomly, and the total number of spare processors equals or exceeds the number of system nodes identified for replacement.

Let F_i be a set of faulty nodes in the system after the i-th application of an Algorithm_1, with F_0 denoting a set of faulty nodes just prior to diagnosis. Let $p_i(F \longrightarrow F')$ be the probability of attaining fault set F' after i applications of Algorithm_1 to F. We seek the probability of repair, P (nf, i), the probability that, starting with fault sets of size nf, there will be no faulty nodes after i applications of Algorithm_1. That is,

$$P(nf,i) = \sum_{|F_0|=nf} p_i(F_0 \longrightarrow N) \qquad \text{(eqn. 4.1)}$$

N : empty (null) set

where the sum is over all fault sets of size nf and where N is the empty (null) set.

$$P(nf,i) = \sum_{|F_0|=nf} \sum_{F_{i-1}} p_{i-1}(F_0 \longrightarrow F_{i-1})\, p(F_{i-1} \longrightarrow N)$$

(eqn. 4.2)

where the left sum is over all initial sets of nf faulty nodes and the right sum is over all sets of faulty nodes. The derivation which follows is for $p(F_{i-1} \longrightarrow F_i)$, where F_{i-1} and F_i are any faulty set. The second factor in

Equation 2 is then obtained directly by a substitution of N for F_i, while the first factor is obtained iteratively.

We derive $p(F_{i-1} \longrightarrow F_i)$ by tracking the number of nodes which undergo various transitions. We consider the derivation of $p(F_{i-1} \longrightarrow F_i)$ for a two node system using Algorithm_1 in next section. For all cases, the approach is to derive two probabilities. That is,

$$p(F_{i-1} \longrightarrow F_i) = \sum_a p(F_{i-1} \longrightarrow a)p(a \longrightarrow F_i) \qquad \text{(eqn. 4.3)}$$

where $p(F_{i-1} \longrightarrow a)$ is the probability F_{i-1} produces syndrome a, $p(a \longrightarrow F_i)$ is the probability a choice of spare nodes, as determined by "a" and Algorithm_1, will produce F_i, and the sum is over all possible syndromes "a". We have,

$$p(F_{i-1} \longrightarrow a) = 2^{-nf'} \qquad \text{(eqn. 4.4)}$$

where $nf = |F_0|$, the number of initially faulty nodes and $nf' = |F_{i-1}|$, the number of faulty nodes after the i-1 th application of the Algorithm_1. $p(a \longrightarrow F_i)$ is just the number of ways to choose spares so that F_i is produced divided by the total number of ways to choose spares.

While we show derivations for certain 2, 3, 4, and 5 node single loop systems, the derivations apply to all designs.

B. A MATRIX REPRESENTATION OF THE PROBABILITY OF REPAIR CALCULATION

We show in this section that the probability of repair as calculated in Section A, can also be computed using matrix operations. This not only gives additional insight into the problem, but also facilitates a calculation of approximate values of the probability of repair. Approximate values are necessary when the number of nodes is such that the calculations (either by Equation 4.3 or matrix manipulations) require inordinately large amounts of computer time and memory. For example, the calculations by Lee [Ref. 19] requires computation times on the order of days for even moderate sizes of nodes (n = 15). Approximate values are calculated in Chapter V. In this section, we illustrate the matrix implementation of Equation 4.3 by the two node system.

1. One application of Algorithm 1

When the test outcome is zero (a_{ba} = 0). Algorithm_1 replaces one good node "b" from 8 good spares. The contribution of this test result to $P_1(F_0 \longrightarrow N)$ is (1/2) (8/10), since there is a probability of 1/2 that a_{ba} = 0 and there are 8 out of 10 ways to choose a fault-free spare. This is shown on the next page. When test outcomes one (a_{ba} = 1), Algorithm_1 replaces two good nodes "a" and "b" from 8 good spares. The contribution of this test result is (1/2) 28/A, where A is the total number of ways to choose spares.

Here, 1/2 is the probability $a_{ba} = 1$, and 28 is the number of ways 2 good nodes can be chosen from 8. A is calculated as shown on these page. Here randomly chosen spares could be all good spares with one possible way or all bad spares with one possible way or one good, one bad spare with two possible ways (A = 61). We calculate the probability of repair after the first application of Algorithm_1 as 0.6294.

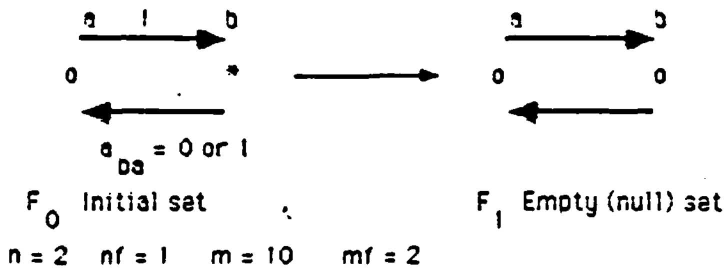


Figure 4.1 Illustration of First Application (i = 1) Using Algorithm_1 for Two Nodes System.

$$P_1(F_0 \rightarrow N) = \frac{1}{2}\left[\frac{\binom{8}{1}}{\binom{10}{1}}\right] + \frac{1}{2}\left[\frac{\binom{8}{2}}{A}\right] = \boxed{0.6294} \qquad \text{(eqn. 4.5)}$$

$$A = \binom{8}{2} 1 + \binom{2}{2} 1 + \binom{8}{1}\binom{2}{1} 2 = 61 \qquad \text{(eqn. 4.6)}$$

To do this we need to know the number of ways to choose k nodes from n. This is,

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!} \qquad \text{(eqn. 4.7)}$$

Let ipj be the probability of transition from state i to state j, where a state is a specification of which nodes are faulty and which are fault-free. Figure 4.1 shows the possible transitions between states, assuming initially that node is faulty.

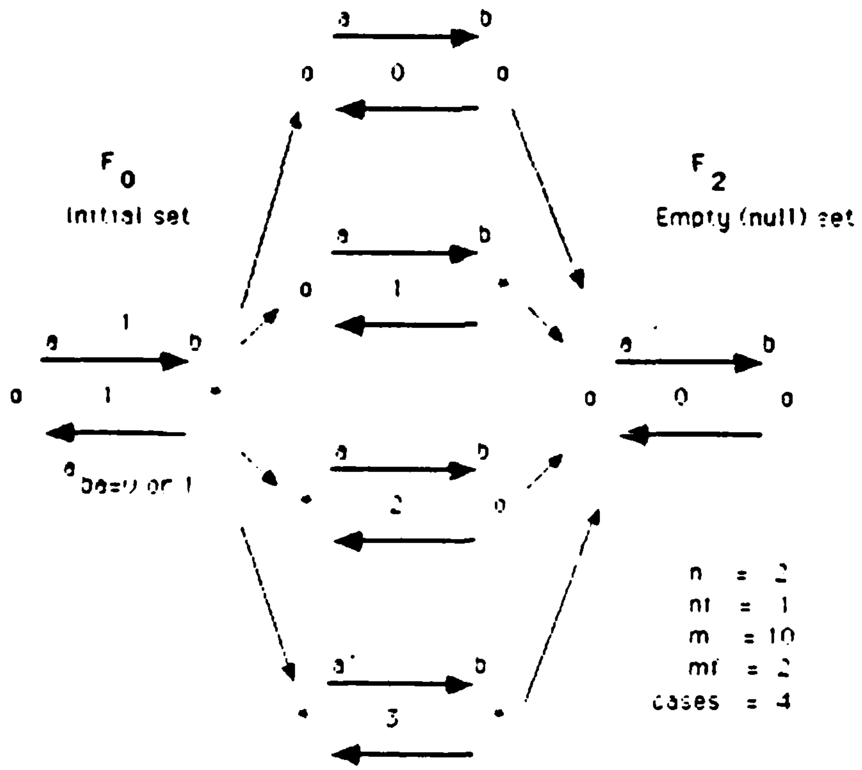


Figure 4.2 Illustration of second application (i = 2) using Algorithm_1 for two nodes system.

2. Two applications of Algorithm 1

Now let's calculate the probability of repair after two applications of the Algorithm_1 for a two node system.

$$1p1 = \frac{1}{2}\left[\overbrace{\frac{\binom{2}{1}}{\binom{10}{1}}}^{a_{ba}=0} + \overbrace{\frac{\binom{8}{1}\binom{2}{1}}{A}}^{a_{ba}=1}\right] = 0.2312 \qquad \text{(eqn. 4.8)}$$

$$1p2 = \frac{1}{2}\left[0 + \frac{\binom{8}{1}\binom{2}{1}}{A}\right] = 0.1312 \qquad \text{(eqn. 4.9)}$$

$$1p3 = \frac{1}{2}\left[0 + \frac{\binom{2}{2}}{A}\right] = 0.0082 \qquad \text{(eqn. 4.10)}$$

$$1p0 = \frac{1}{2}\left[\frac{\binom{8}{1}}{\binom{10}{1}} + \frac{\binom{8}{2}}{A}\right] = 0.6294 = 2p0 \qquad \text{(eqn. 4.11)}$$

$$3p0 = \frac{1}{4}\left[\underbrace{0}_{\substack{a_{ab}=0\\ a_{ba}=0}} + \underbrace{0}_{\substack{a_{ab}=0\\ a_{ba}=1}} + \underbrace{0}_{\substack{a_{ab}=1\\ a_{ba}=0}} + \underbrace{\frac{\binom{9}{2}}{B}}_{\substack{a_{ab}=1\\ a_{ba}=1}}\right] = 0.1666 \qquad \text{(eqn. 4.12)}$$

$$
\begin{array}{llcccc}
 & & \underline{i=1} & & \underline{i=2} & \underline{\text{states}} \\
P(F_2 \rightarrow N_0) = & \Big[& 0.6294 & \times & 1.0 & 0 \\
 & + & 0.2312 & \times & 0.6294 & 1 \\
 & + & 0.1312 & \times & 0.6294 & 2 \\
 & + & 0.0082 & \times & 0.1666 \; \Big] & 3 \\
 & = & \boxed{0.8589} & & &
\end{array}
\qquad \text{(eqn. 4.13)}
$$

We calculate 1p0 = 0.6294 which is the probability of repair from state 1 to state 0. Also 0p0 = 1.0 since Algorithm_1 leaves the system unchanged if it is in state 0.

The calculation of 1p1:

When test outcomes aba = 0, Algorithm_1 replaces one bad node "b" from 2 bad spares. When test outcomes aba = 1, Algorithm_1 replaces one good node "a" from 8 good spares, one bad node "b" from 2 bad spares. As shown in Figure 4.4, the result is 1p1 = 0.2312.

The calculation of 1p2:

When test outcomes aba = 0, the probability of transition is 0 because Algorithm_1 does not replace node "a". So node "a" does not change from good to bad. When the test outcomes aba = 1 Algorithm_1 replaces one good node "b" from 8 good spares one bad node "a" from 2 bad spares. The result is 1p2 = 0.1312.

The calculation of 1p3:

When test outcomes a_{ba} = 0, the probability of transition is 0 (likewise calculation of 1p2). When test outcomes a_{ba} = 1 Algorithm_1 replaces two bad nodes "a" and "b" from 2 bad spares. The result is 1p3 = 0.0082.

The calculation of 1p0:

When test outcomes a_{ba} = 0 Algorithm_1 replaces one good node "b" from 8 good spares. When test outcomes a_{ba} = 1 Algorithm_1 replaces two good node "a" and "b" from 8 good spares. The result is 1p0 = 0.6294.

The calculation of 2p0:

The result is same as 1p0. Since state 1 and state 2 are the same except for a rearrangement of faulty nodes. (Assumption 1). Therefore, 2p0 = 1p0 = 0.6294.

The calculation of 3p0:

Unlike the cases discussed previously, Algorithm_1 may fail to replace any node. That is, when both faulty nodes produce pass test outcomes Algorithm_1 makes no replacement. When a_{ba} = a_{ab} = 1, the transition to state 0 requires that Algorithm_1 replace both units with fault-free spares. This is with probability 0.1666. When a_{ab} = 0 and a_{ba} = 1 or vice versa only one faulty node is replaced. For these test outcomes attaining state 0 is impossible. Thus, the total probability of transition from state 3 to 0 is 0.1666. The number of ways, B, to make choices from spares is 54.

3. <u>Three applications of Algorithm 1</u>

Now let's consider three applications (i = 3) of Algorithm_1 for two nodes system.

We can realize that this calculation will be just a repetition of the calculation in Figure 4.2. This is exactly same transition from one state to another state.

We already calculated first row and second column of transition matrix P before. Note $_0p_1 = {_0p_2} = {_0p_3} = 0$, because if the initial state is 0, the final state will also be 0. Because of symmetry $_1p_1 = {_2p_2}$, $_1p_2 = {_2p_1}$, $_1p_3 = {_2p_3}$ and $_3p_1 = {_3p_2}$. (Assumtion 1)

The calculation of $_3p_1$:

Here nf' = 2 when test outcomes a_{ba} = 0. There is no possibility, because Algorithm_1 does not replace node "a" from bad to good. When a_{ab} = 0 and a_{ba} = 1, Algorithm_1 replaces one good node "a" from 9 good spares. When a_{ab} = a_{ba} = 1, Algorithm_1 replaces one good node "a" from 9 good spares, one node "b" from 1 bad spares. The result is $_3p_1$ = 0.2667.

The calculation of $_3p_2$:

The result will be same as $_3p_1$. This is another arrangement of $_3p_1$. There is no difference (Assumption 1). So $_3p_2$ = $_3p_1$ = 0.2667.

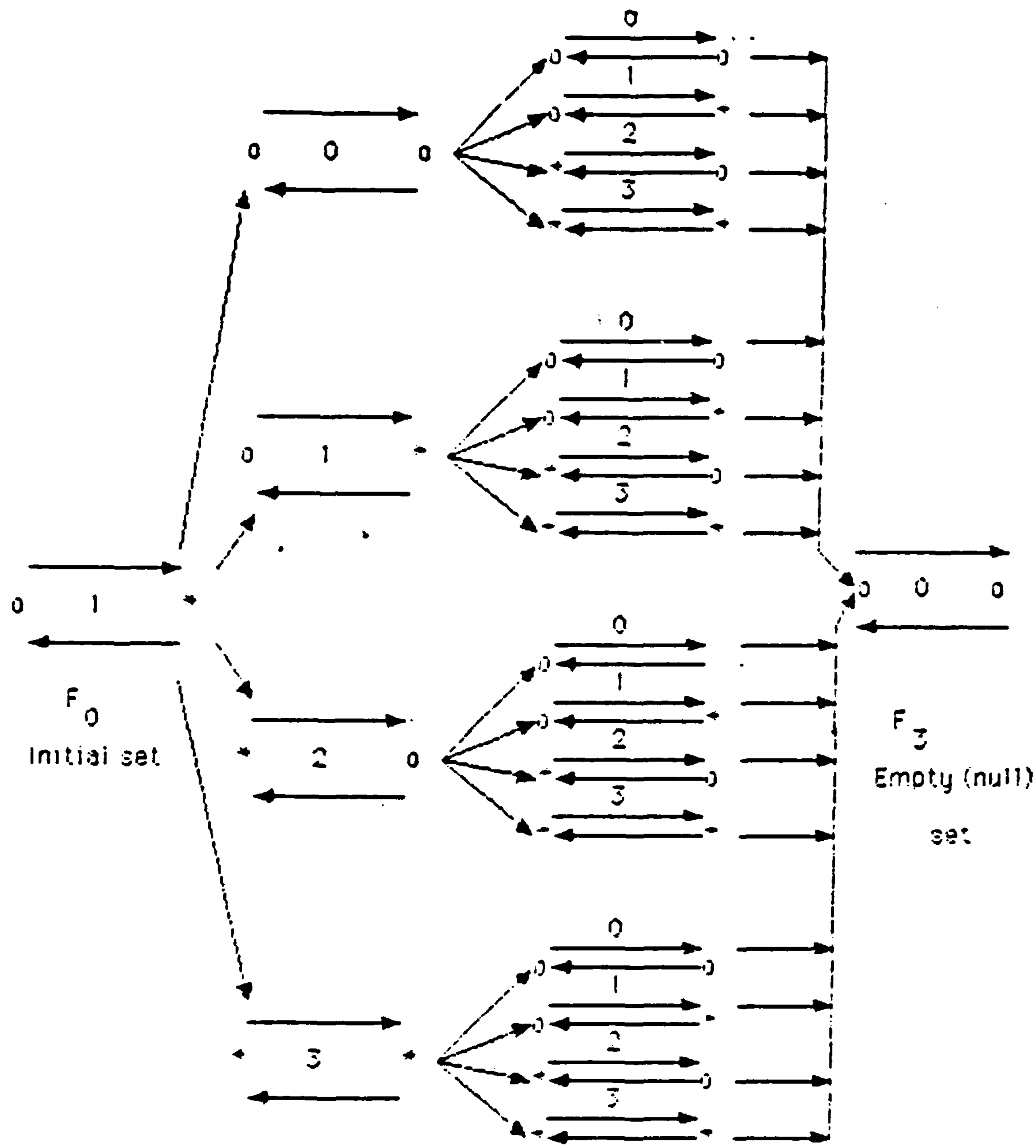


Figure 4.3 Illustration of third application (i = 3) using Algorithm_1 for two nodes system.

The calculation of 3p3:

When test outcomes $a_{ab} = a_{ba} = 0$, there is no replacement and the probability is 1.0. When $a_{ba} = 1$ Algorithm_1 replaces one bad node "b" from one bad spare when $a_{ab} = 1$ Algorithm_1 replaces one bad node "a" from one bad spare. When $a_{ab} = a_{ba} = 1$, the probability of transition is 0. Because the system has only one bad spares. Therefore Algorithm_1 does not replace two bad nodes "a" and "b" from one bad spare. The result is 3p3 = 0.3.

$$
\begin{array}{cccc}
a_{ab} = a_{ba} = 0 & a_{ab} = 1\ a_{ba} = 0 & a_{ab} = 0\ a_{ba} = 1 & a_{ab} = a_{ba} = 1
\end{array}
$$

$$3p1 = \frac{1}{4}\left[\,0 + 0 + \frac{\binom{9}{1}}{\binom{10}{1}} + \frac{\binom{9}{1}\binom{1}{1}}{B}\right] = 0.2667 \quad \text{(eqn. 4.14)}$$

$$3p2 = \frac{1}{4}\left[\,0 + \frac{\binom{9}{1}}{\binom{10}{1}} + 0 + \frac{\binom{9}{1}\binom{1}{1}}{B}\right] = 0.2667 \quad \text{(eqn. 4.15)}$$

$$3p3 = \frac{1}{4}\left[\,1 + \frac{\binom{1}{1}}{\binom{10}{1}} + \frac{\binom{1}{1}}{\binom{10}{1}} + \frac{\cancelto{0}{\binom{1}{2}}}{B}\right] = 0.3 \quad \text{(eqn. 4.16)}$$

$$B = \underset{\text{all good}}{\binom{9}{2}\ 1\ \text{way}} + \underset{\text{all bad}}{\cancelto{0}{\binom{1}{2}}\ 1\ \text{way}} + \underset{\text{one good one bad}}{\binom{9}{1}\binom{1}{1}\ 2\ \text{ways}} = 54 \quad \text{(eqn. 4.17)}$$

We noticed that each application of Algorithm_1 has the same transition states. Therefore the P matrix is called a transition matrix of the probability of repair.

$$P = \begin{bmatrix} 0p0 & 1p0 & 2p0 & 3p0 \\ 0p1 & 1p1 & 2p1 & 3p1 \\ 0p2 & 1p2 & 2p2 & 3p2 \\ 0p3 & 1p3 & 2p3 & 3p3 \end{bmatrix}_{4\times 4} = \begin{bmatrix} 1 & 0.6294 & 0.6294 & 0.1666 \\ 0 & 0.2312 & 0.1312 & 0.2667 \\ 0 & 0.1312 & 0.2312 & 0.2667 \\ 0 & 0.0082 & 0.0082 & 0.3000 \end{bmatrix}_{4\times 4}$$

(eqn. 4.18)

V. CHARACTERIZATION OF APPROXIMATION USING MATRICES

A. FIRST APPROXIMATION

While the matrix multiplications for the examples in Chapter IV can be done easily by hand or computer, the multiplications increase dramatically as the number of nodes increases. For an n node system, the transition matrix dimension is $2^n \times 2^n$ as shown in Table 1. Its size is very large for even moderate values of n.

We illustrate our method of approximation using the two node system. For example, if we have two nodes (n = 2), we get four states (2^2) indexed by s = 0, 1, 2, 3.

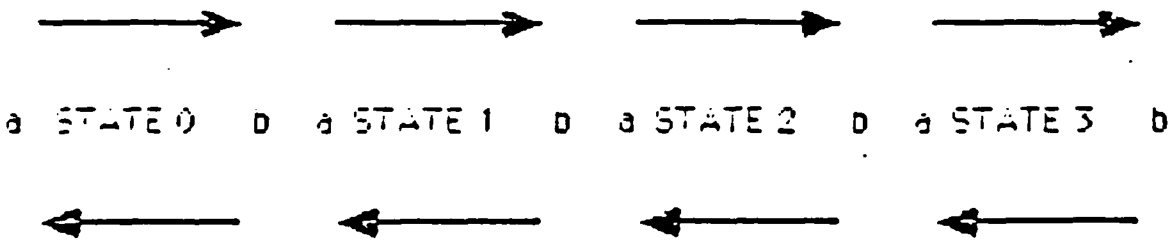


Figure 5.1 Four states to repair the two nodes system

Let P_s be the probability of being in state s before diagnosis and P_s' is the probability of being in state s after diagnosis. Therefore, after one application of Algorithm_1, we have,

$$P\ X = X' \qquad \text{(eqn. 5.1)}$$

Now we seek,

$$P^k X = X' \qquad \text{(eqn. 5.2)}$$

where P^k is probability of transition states after k applications of the diagnostic algorithm.

$$
\begin{aligned}
P_0' &= P_0\,0p0 + P_1\,1p0 + P_2\,2p0 + \ldots + P_s\,sp0 \\
P_1' &= P_0\,0p1 + P_1\,1p1 + P_2\,2p1 + \ldots + P_s\,sp1 \\
&\vdots \\
P_s' &= P_0\,0ps + P_1\,1ps + P_2\,2ps + \ldots + P_s\,sps
\end{aligned}
\qquad \text{(eqn. 5.3)}
$$

This can be expressed as a matrix multiplication.

$$
\underbrace{\begin{bmatrix} 0p0 & 1p0 & 2p0 & \ldots & sp0 \\ 0p1 & 1p1 & 2p1 & \ldots & sp1 \\ \vdots & \vdots & \vdots & & \vdots \\ 0ps & 1ps & 2ps & \ldots & sps \end{bmatrix}}_{P}
\underbrace{\begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_s \end{bmatrix}}_{X}
=
\underbrace{\begin{bmatrix} P_0' \\ P_1' \\ \vdots \\ P_s' \end{bmatrix}}_{X'}
\qquad \text{(eqn. 5.4)}
$$

Markov chains provide one example in which we are interested in computing $P^k X$. The transition matrix P has the property that all entries are nonnegative and the sum of the entries in any column is 1. X is the probability transition vector before diagnosis. Then PX is the corresponding probability transition vector after one application of the diagnostic algorithm. Similarly, $P^2 X$ is the probability transition vector after two applications of the diagnostic algorithm and, in general, $P^k X$ gives the probability transition vector after k applications of the diagnostic algorithm.

In order to derive a simple closed form approximation to the probability of repair, it is necessary to derive the eigenvalues of the transition matrix.

<u>Note</u>: Suppose that P is an nxn matrix and V is a column vector with n components such that;

$$P\ V = E\ V \qquad \text{(eqn. 5.5)}$$

for some scalar E, where E is eigenvalue, V is eigenvector. It is an easy exercise to show that;

$$P^k V = E^k V \qquad \text{(eqn. 5.6)}$$

Thus $P^k X$ is easily computed if X is equal to this vector V.

1. Diagonalizing a matrix

A square matrix is called diagonal if all nondiagonal entries are zero. Later we will indicate the importance of being able to compute $P^k X$. We show that if P has s distinct eigenvalues, then P^k in this computation of $P^k X$ can be essentially replaced by D^k, where D is a diagonal matrix with the eigenvalues of P as diagonal entries. It can be seen that D^k is the diagonal matrix obtained from D by raising each diagonal entry to the power of k.

THEOREM: (Diagonalization)

Let P be an nxn matrix having n distinct eigenvalues E_0, E_1, E_2, ..., E_s. Let V_0, V_1, V_2, ..., V_s be column vectors in R^n such that V_i is an eigenvector corresponding to E_i for i = 0, 1, 2, ..., s. Set C be the nxn matrix having V as i-th column vector. Then C is invertible and C^{-1} P C is equal to the diagonal matrix D.

$$D = \begin{bmatrix} E_0 & & & \\ & E_1 & & 0 \\ & & \ddots & \\ 0 & & & E_s \end{bmatrix} \qquad \text{(eqn. 5.7)}$$

Proof:

We know that,

$$P\ V_i = E_i\ V_i \qquad \text{(eqn. 5.8)}$$

So PC = CD and C^{-1} P C = D. We have,

$$CD = \begin{bmatrix} | & | & | & & | \\ V_0 & V_1 & V_2 & \cdots\cdots & V_s \\ | & | & | & & | \end{bmatrix} \begin{bmatrix} E_0 & & & \\ & E_1 & & \bigcirc \\ & & \ddots & \\ \bigcirc & & & E_s \end{bmatrix}$$

(eqn. 5.9)

$$CD = \begin{bmatrix} | & | & | & & | \\ E_0 V_0 & E_1 V_1 & E_2 V_2 & \cdots\cdots\cdots & E_s V_s \\ | & | & | & & | \end{bmatrix}$$

Now we show how the computation of P^k can be essentially reduced to the computation of D^k. Let P , E_i , V_i , and C be as in Theorem. Then $P^k = C D^k C^{-1}$ for each positive integer k. From C^{-1} P C = D, we obtain P = C D C^{-1}. Thus,

$$P^k = \underbrace{(CDC^{-1})(CDC^{-1})\ldots\ldots(CDC^{-1})}_{k \text{ factors}} \qquad \text{(eqn. 5.10)}$$

Clearly the adjacent terms C^{-1} C cancel. We get $P^k = C D^k C^{-1}$. Then we say that P has been diagonalized by C.

Q.E.D.

An nxn matrix with n distinct eigenvalues is similar to a diagonal matrix. It is not always essential that the eigenvalues be distinct.

2. <u>Computing $P^k X$</u>

Let P be a diagonalizable nxn matrix and let E_0, E_1, E_2, ..., E_s be the (not necessarily distinct) eigenvalues of P. Let V_0, V_1, V_2, ..., V_s be a basis for R^n, where V_i is an eigenvector for E_i. We have seen that if C is the matrix having V_i as the i th column vector, then

$$C^{-1} P C = D = \begin{bmatrix} E_0 & & & 0 \\ & E_1 & & \\ & & \ddots & \\ 0 & & & E_s \end{bmatrix} \qquad \text{(eqn. 5.11)}$$

Let X be any vector in $\mathbb{R}^n$. We show that from Equation 5.6 we get,

$$P^k X = C D^k C^{-1} X = \begin{bmatrix} | & | & & | \\ E_0^k V_0 & E_1^k V_1 & \cdots & E_s^k V_s \\ | & | & & | \end{bmatrix} C^{-1} X \qquad \text{(eqn. 5.12)}$$

Now $C^{-1} X$ is a column vector. It may be any column vector in R^n. We set,

$$C^{-1}X = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_s \end{bmatrix} = d \qquad \text{(eqn. 5.13)}$$

Then, we form

$$P^k X = d_0 E_0^k V_0 + d_1 E_1^k V_1 + \dots\dots\dots + d_s E_s^k V_s . \qquad \text{(eqn. 5.14)}$$

This equation expresses $P^k X$ as a linear combination of the eigenvectors V_i. X is a vector whose components are the probability of being in a particular state of the system. The probability vector at the next stage of the process is found by multiplying the present probability vector on the left by a transition matrix P. Illustrations of this situation are provided by Markov chains and the generation of the Fibonacci sequence.

We are interested in the long term outcome of the process. That is, we wish to study $P^k X$ for large values of k. In particular, suppose that ($|E_0| > |E_1|$) so that E is the unique eigenvalue of maximum magnitude. If $d_0 \neq 0$

and k is large, the vector $P^k X$ is approximately $d_1 \; E_1{}^k \; V_1$ in the sense that $\| P^k X - d_0 \; E_0{}^k \; V_0 \|$ is small compared with $\| P^k X \|$.

Let's consider our example again. We can get eigenvalues and eigenvectors from MATLAB computer program for two node system. PC-MATLAB is a general program for the scientific and engineering numeric calculations. The name MATLAB stands for "MATrix LABoratory". PC-MATLAB requires the following hardware and software:

- IBM PC, PC/XT, PC/AT, or compatible MS-DOS computer.
- At least 320K of memory.
- MS-DOS Version 2.0 or higher.
- 8087 numeric coprocessor chip.
- At least one DSDD floppy disk drive.

$$\mathbf{C} = \begin{bmatrix} 1 & -0.0000 & 1.0000 & 1.0000 \\ 0 & +1.0000 & -0.4636 & -0.6216 \\ 0 & -1.0000 & -0.4636 & -0.6216 \\ 0 & -0.0000 & -0.0729 & 0.2431 \end{bmatrix}_{4 \times 4} \qquad \text{(eqn. 5.15)}$$

$$\mathbf{D} = \begin{bmatrix} 1 & & & \\ & 0.1000 & & \\ & & 0.4043 & \\ & & & 0.2581 \end{bmatrix}_{4 \times 4} \qquad \text{(eqn. 5.16)}$$

$$C^{-1} = \begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 \\ 0 & +0.5000 & -0.5000 & -0.0000 \\ 0 & -0.7694 & -0.7694 & -3.9338 \\ 0 & -0.2306 & -0.2306 & 2.9338 \end{bmatrix}_{4\times 4} \qquad \text{(eqn. 5.17)}$$

Substituting these eigenvalues into Equation 5.14,

$$P^k X = d_0(1)^k V_0 + d_1(0.1)^k V_1 + d_2(0.4043)^k V_2 + d_3(0.2581)^k V_3$$

(eqn. 5.18)

Then,

$$P^k X - d_0 V_0 = d_1 (0.1)^k V_1 + d_2(0.4043)^k V_2 + d_3 (0.2581)^k V_3$$

(eqn. 5.19)

As k increase, the middle term on the right dominates and as an approximation, we have

$$P^k X - d_0 V_0 = d_2 (0.4043)^k V_2 \qquad \text{(eqn. 5.20)}$$

From our definition

$$P^{k}X = X' = \begin{bmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \end{bmatrix} \qquad \text{(eqn. 5.21)}$$

where X' vector is the probability of being in each states s after diagnosis.

$$C^{-1}X = d \qquad \text{(eqn. 5.22)}$$

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 \\ 0 & +0.5000 & -0.5000 & -0.0000 \\ 0 & -0.7594 & -0.7594 & -3.9338 \\ 0 & -0.2306 & -0.2306 & 2.9338 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} \qquad \text{(eqn. 5.23)}$$

We have,

$$d_0 = P_0 + P_1 + P_2 + P_3 = 1 \qquad \text{(eqn. 5.24)}$$

Also,

$$V_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4 \times 1} \qquad \text{(eqn. 5.25)}$$

and arranging our Equation 5.10. We get,

$$\begin{bmatrix} P_0^{\cdot} \\ P_1^{\cdot} \\ P_2^{\cdot} \\ P_3^{\cdot} \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = d_2 (0.4043)^k V_2 \qquad \text{(eqn. 5.26)}$$

We get d_2 from Equation 5.23 and V_2 third column vector from C matrix. So that,

$$d_2 = 0.0\ P_0 - 0.7694 P_1 - 0.7694 P_2 - 3.9338 P_3 \qquad \text{(eqn. 5.27)}$$

$$V_2 = \begin{bmatrix} 1.0000 \\ -0.4636 \\ -0.4636 \\ -0.0729 \end{bmatrix}_{4\times 1} \qquad \text{(eqn. 5.28)}$$

We define,

$P_0 = 1 \quad P_1 = P_2 = P_3 = 0 \longrightarrow$ 1st column of P^k

$P_1 = 1 \quad P_0 = P_2 = P_3 = 0 \longrightarrow$ 2nd column of P^k

$P_2 = 1 \quad P_0 = P_1 = P_3 = 0 \longrightarrow$ 3rd column of P^k

$P_3 = 1 \quad P_0 = P_1 = P_2 = 0 \longrightarrow$ 4th column of P^k

Then we find all computations of P^k

$$\begin{bmatrix} P_0^{\cdot}-1 \\ P_1^{\cdot} \\ P_2^{\cdot} \\ P_3^{\cdot} \end{bmatrix}_{4\times 1} = \begin{bmatrix} -0.7694P_1 & -0.7694P_2 & -3.9338P_3 \end{bmatrix}_{1\times 4} (0.4043)^k \begin{bmatrix} 1.0000 \\ -0.4636 \\ -0.4636 \\ -0.0729 \end{bmatrix}_{4\times 1}$$

(eqn. 5.29)

$$\begin{bmatrix} P_0^{\cdot}-1 \\ P_1^{\cdot} \\ P_2^{\cdot} \\ P_3^{\cdot} \end{bmatrix} = \begin{bmatrix} 0 & -0.7694(0.4043)^k & -0.7694(0.4043)^k & -3.9338(0.4043)^k \\ 0 & 0.3567(0.4043)^k & 0.3567(0.4043)^k & 1.8237(0.4043)^k \\ 0 & 0.3567(0.4043)^k & 0.3567(0.4043)^k & 1.8257(0.4043)^k \\ 0 & 0.0561(0.4043)^k & 0.0561(0.4043)^k & 0.2368(0.4043)^k \end{bmatrix}$$

(eqn. 5.30)

$$P^k = \begin{bmatrix} P_0^{\cdot} \\ P_1^{\cdot} \\ P_2^{\cdot} \\ P_3^{\cdot} \end{bmatrix} = \begin{bmatrix} 1 & 1-0.7694(E_1)^k & 1-0.7694(E_1)^k & 1-3.9338(E_1)^k \\ 0 & 0.3567(E_1)^k & 0.3567(E_1)^k & 1.8237(E_1)^k \\ 0 & 0.3567(E_1)^k & 0.3567(E_1)^k & 1.8237(E_1)^k \\ 0 & 0.0561(E_1)^k & 0.0561(E_1)^k & 0.2368(E_1)^k \end{bmatrix}$$

(eqn. 5.31)

P^k This vector is probability of transition states after k applications of the diagnostic algorithm I.

$$P_0^{\cdot} = [1.0(P_0+P_1+P_2+P_3)] - 0.7694(E_1)^k(P_1+P_2) - 3.9338(E_1)^k P_3$$

$$P_1^{\cdot} = P_2^{\cdot} = 0.3567(E_1)^k(P_1+P_2) + 1.8237(E_1)^k P_3$$

$$P_3^{\cdot} = 0.0561(E_1)^k(P_1+P_2) + 0.2368(E_1)^k P_3 \qquad \text{(eqn. 5.32)}$$

$E_1 = 0.4043$ Second largest eigenvalue

B. COMPUTING SECOND LARGEST EIGENVALUE BY THE POWER METHOD

As shown in Equation 5.10, the second largest eigenvalue must be calculated. Several algorithms have been developed for calculating the largest (or smallest) eigenvalue, and no one method can be considered the best for all cases. Computation of the eigenvalues of a matrix is one of the toughest jobs in linear algebra. We use the program MATLAB in this thesis. There are other methods; for example, the power method, Jacobi's method, and the QR method.

We also use the power method. It is especially useful if one wants only the eigenvalue of largest (or of smallest) magnitude, as in many vibration problems. So we need second largest eigenvalue here. We know that our first largest eigenvalue is always 1.

<u>Definition</u>: An eigenvalue of a matrix P is called the dominant eigenvalue of P if its absolute value is larger than the absolute values of the remaining eigenvalues. An eigenvector corresponding to the dominant eigenvalue is called a dominant eigenvector of P.

Since P is diagonalizable, there exists a basis $\{V_0, V_1, \ldots, V_s\}$ for $\mathbb{R}^n$ composed of eigenvectors of P. We assume that V_1 is the eigenvector corresponding to E_1, and

that the numbering is such that $|E_0| > |E_1| \geq |E_2| \geq \ldots\ldots \geq |E_s|$. Let W be any nonzero vector in $\mathbb{R}^n$. Also the sum of the values in W is 1. Then we can approximate an eigenvector of P corresponding to the dominant eigenvalue E_1 by multiplying an appropriate initial approximation vector W repeatedly by P.

Repeated multiplication of W by P may produce very large (or very small) numbers. It is customary to scale after each multiplication to keep the components of the vectors at a reasonable size. After the first multiplication, we find the maximum m of the magnitudes of all the components of PW and apply P next time to the vector $W_1 = (1/m)PW$. Similarly, we let $W_2 = (1/m_1)PW_1$, where m_1 is the maximum of the magnitudes of components of PW_1, and so on. We summarize the steps of the Power method in the following Table 2.

TABLE 2

THE POWER METHOD FOR FINDING THE SECOND LARGEST EIGENVALUE OF P

Step 1	Choose an appropriate vector W in $\mathbb{R}^n$ as first approximation to an eigenvector for E_1.
Step 2	Compute PW and the Rayleigh quotient (PW . W)/(W . W).
Step 3	Let $W_1 = (1/m)PW$, Where m is the maximum of the magnitudes of components of PW.
Step 4	Go to step 2, and repeat with all subscripts increased by 1. The Rayleigh quotients should approach E^1, and the W_j should approach an eigenvector of P corresponding to E_1.

We find second largest eigenvalue for 2, 3, 4 and 5 nodes system by power method in the following pages.

Note that if P has an eigenvalue of nonzero magnitude smaller than that of any other eigenvalue, then the power method can be used with P^{-1} to find this smallest eigenvalue. Recall that the eigenvalues of P^{-1} are the reciprocals of the eigenvalues of P, and the eigenvectors are the same.

Selected arbitrary matrix W to get second largest eigenvalue of the (1/2 nodes bad and 2/10 spares bad) system.

$$\text{ARBITRARY VECTOR } W = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}_{4x1} \qquad \text{(eqn. 5.33)}$$

TABLE 3

POWER METHOD FOR TWO NODE SYSTEM

	Rayleigh Quotient
1. step of power method	0.3334
2. " "	0.5256
3. " "	0.4718
4. " "	0.4423
5. " "	0.4267
6. " "	0.4179
7. " "	0.4128
8. " "	0.4096
9. " "	0.4077
10. " "	0.4064
11. " "	0.4057
12. " "	0.4952
13. " "	0.4049
14. " "	0.4047
15. " "	0.4045
16. " "	0.4044
17. " "	0.4043 ->second largest eigenvalue

Note: After 17 steps of the power method, we get our second largest eigenvalue.

Selected arbitrary matrix W to get second largest eigenvalue of the (2/3 nodes bad and 2/10 spares bad) system.

$$\text{ARBITRARY VECTOR } W = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}_{8x1} \qquad \text{(eqn. 5.34)}$$

TABLE 4

POWER METHOD FOR THREE NODE SYSTEM

	Rayleigh Quotient
1. step of power method	0.2309
2. " "	0.5084
3. " "	0.5661
4. " "	0.5481
5. " "	0.5305
6. " "	0.5216
7. " "	0.5178
8. " "	0.5161
9. " "	0.5155
10. " "	0.5152
11. " "	0.5151
12. " "	0.5150 -> second largest eigenvalue

Note: After 12 steps of the power method, we get our second largest eigenvalue.

Selected arbitrary matrix W to get second largest eigenvalue of the (3/4 nodes and 2/10 spares bad) system.

$$\text{ARBITRARY VECTOR } W = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \end{bmatrix}_{5\times 1} \qquad \text{(eqn. 5.35)}$$

TABLE 5

POWER METHOD FOR FOUR NODE SYSTEM

	Rayleigh Quotient
1. step of power method	0.2004
2. " "	0.7859
3. " "	0.7034
4. " "	0.6552
5. " "	0.6353
6. " "	0.6277
7. " "	0.6248
8. " "	0.6238
9. " "	0.6234
10. " "	0.6232 -> second largest eigenvalue

Note: After 10 steps of the power method, we get our second largest eigenvalue.

Selected arbitrary matrix W to get second largest eigenvalue of the (4/5 nodes bad and 2/10 spares bad) system.

$$\text{ARBITRARY VECTOR } W = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}_{6x1} \qquad \text{(eqn. 5.36)}$$

TABLE 6

POWER METHOD FOR FIVE NODE SYSTEM

	Rayleigh Quotient
1. step of power method	0.1626
2. " "	0.8461
3. " "	0.8538
4. " "	0.7986
5. " "	0.7605
6. " "	0.7401
7. " "	0.7305
8. " "	0.7262
9. " "	0.7243
10. " "	0.7235
11. " "	0.7231
12. " "	0.7230
13. " "	0.7229
14. " "	0.7228 ->second largest eigenvalue

Note: After 14 steps of the power method, we get our second largest eigenvalue.

VI. AGGREGATION OF THE TRANSITION MATRIX

Already we have transition matrix P for two nodes system which is:

$$P = \begin{bmatrix} 0p0 & 1p0 & 2p0 & 3p0 \\ 0p1 & 1p1 & 2p1 & 3p1 \\ 0p2 & 1p2 & 2p2 & 3p2 \\ 0p3 & 1p3 & 2p3 & 3p3 \end{bmatrix}_{4\times4} = \begin{bmatrix} 1 & 0.6294 & 0.6294 & 0.1666 \\ 0 & 0.2312 & 0.1312 & 0.2667 \\ 0 & 0.1312 & 0.2312 & 0.2667 \\ 0 & 0.0082 & 0.0082 & 0.3000 \end{bmatrix}_{4\times4}$$

(eqn. 6.1)

We can illustrate our original transition matrix P as in Figure 6.1

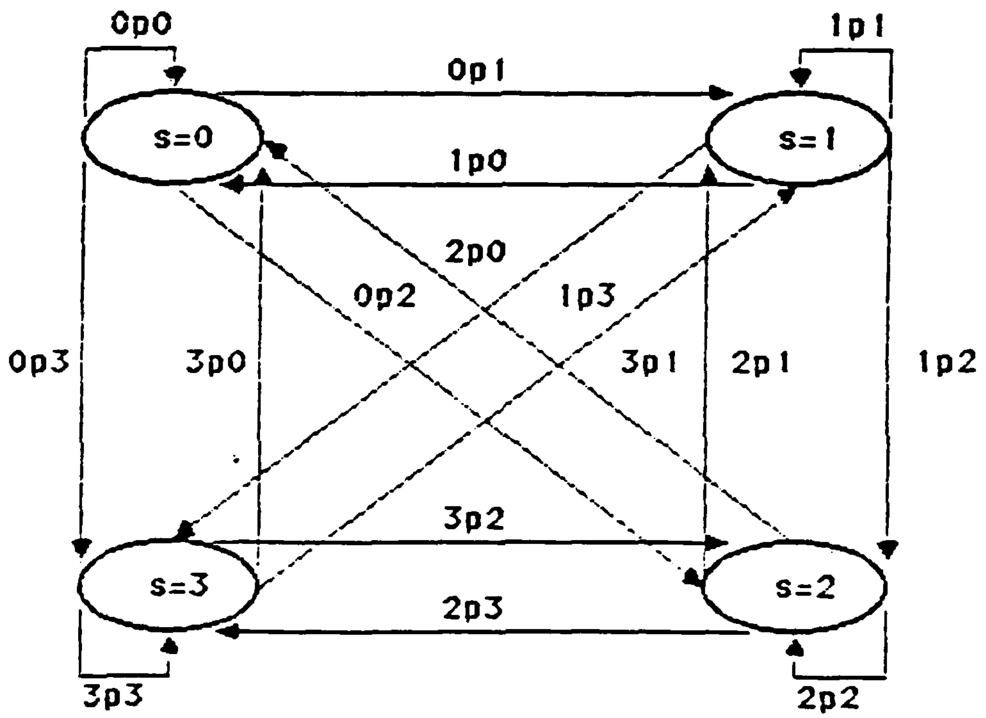


Figure 6.1 Illustration transition states of P

The asymptotic approximation to the probability of repair requires eigenvalues which in turn are produced by our matrix manipulation program MATLAB. However, there is a limit on the size of matrices which MATLAB can handle. With the probability of repair problem as stated, an n node system would require a matrix of dimension $2^n \times 2^n$, which is very large for even moderate size n. To handle larger systems, we choose to represent transition probabilities in a different way which is called AGGREGATION.

Let P be the transition matrix between specific states. Let Pa be the transition matrix between aggregated states. We seek the elements of Pa as a function of the elements of P. Consider Figure 6.2, which shows the transition between all states in the system with q (initial) faulty nodes to all states with p (final) faulty nodes.

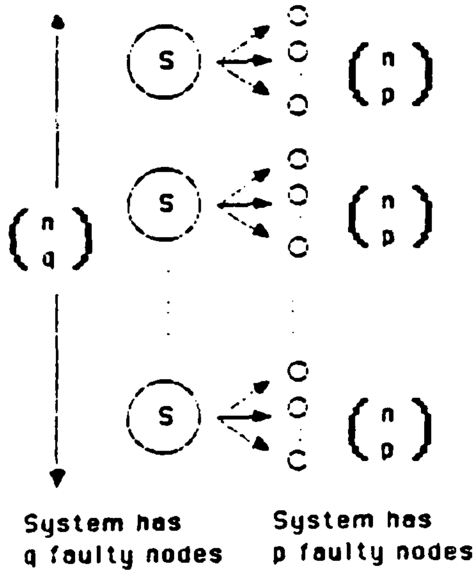


Figure 6.2 Aggregation of system

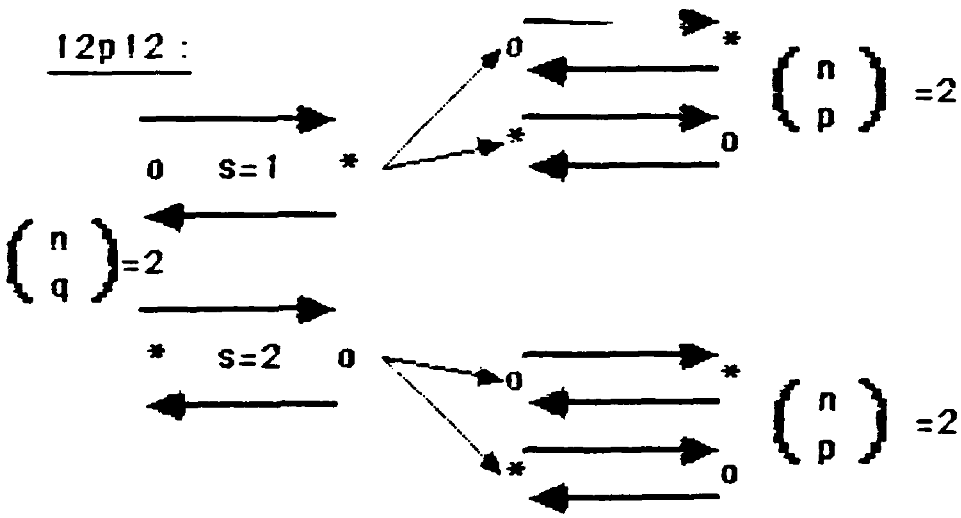
$$\frac{\displaystyle\sum}{\binom{n}{q}\cancel{\binom{n}{p}}}\cancel{\binom{n}{p}} = \frac{\displaystyle\sum}{\binom{n}{q}} = \frac{\cancel{\binom{n}{q}}\textstyle\sum}{\cancel{\binom{n}{q}}} = \textstyle\sum \qquad \text{(eqn 6.1)}$$

$$\displaystyle\sum = \binom{n}{q}\textstyle\sum \qquad \text{(eqn 6.2)}$$

$\displaystyle\sum$: Summation of all possible probabilities from initial state with q faulty nodes to final state with p faulty nodes.

$\textstyle\sum$: Summation of probabilities from one state with q faulty nodes to another states with p faulty nodes over all possible probabilities.

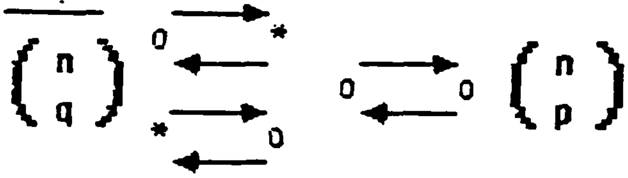
Let's go back to our example for two nodes system. Now we calculate that aggregated transition probability.



$$12p12=\frac{1p1+1p2+2p1+2p2}{\binom{n}{q}}=\frac{0.2312+0.1312+0.1312+0.2312}{2}=0.3624$$

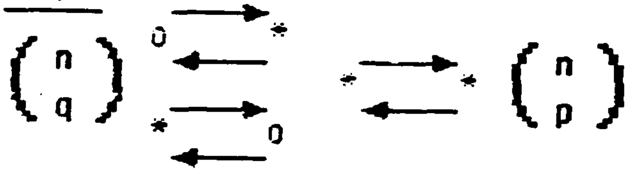
(eqn. 6.2)

12p0:



$$12p0=\frac{1p0+2p0}{\binom{n}{q}}=\frac{0.6294+0.6294}{2}=0.6294 \qquad \text{(eqn. 6.3)}$$

12p3:



$$12p3=\frac{1p3+2p3}{\binom{n}{q}}=\frac{0.0082+0.0082}{2}=0.0082 \qquad \text{(eqn. 6.4)}$$

(eqn. 6.5)

$$Pa=\begin{bmatrix} 0p0 & 12p0 & 3p0 \\ 0p12 & 12p12 & 3p12 \\ 0p3 & 12p3 & 3p3 \end{bmatrix}=\begin{bmatrix} 1 & 0.6294 & 0.1666 \\ 0 & 0.3624 & 0.5334 \\ 0 & 0.0082 & 0.3000 \end{bmatrix}_{3\times3}$$

0p12:

$$\binom{n}{q} \; {}_{0}\!\rightleftarrows\!{}_{0} \; \begin{matrix} {}_{0}\!\rightleftarrows\!{}^{*} \\ {}^{*}\!\rightleftarrows\!{}_{0} \end{matrix} \; \binom{n}{p}$$

$$0p12 = \frac{0p1+0p2}{\binom{n}{q}} = \frac{0.0+0.0}{1} = 0.0 \qquad \text{(eqn. 6.6)}$$

3p12:

$$\binom{n}{q} \; {}_{*}\!\rightleftarrows\!{}_{*} \; \begin{matrix} {}_{0}\!\rightleftarrows\!{}^{*} \\ {}^{*}\!\rightleftarrows\!{}_{0} \end{matrix} \; \binom{n}{p}$$

$$3p12 = \frac{3p1+3p2}{\binom{n}{q}} = \frac{0.2667+0.2667}{1} = 0.5334 \qquad \text{(eqn. 6.7)}$$

Thus, instead of a matrix of dimension $2^n \times 2^n$, we need only matrices of dimension $(n+1) \times (n+1)$.

Using MATLAB program we get our new eigenvalues and new eigenvectors from aggregated transition matrix Pa. Then we get same exact probabilities as given in Appendix A. And using Equation 5.17 we have approximated probabilities as following Equation 6.3.

$$p^k = 1 - 0.7694\,(0.4043)^k \qquad \text{(eqn. 6.8)}$$

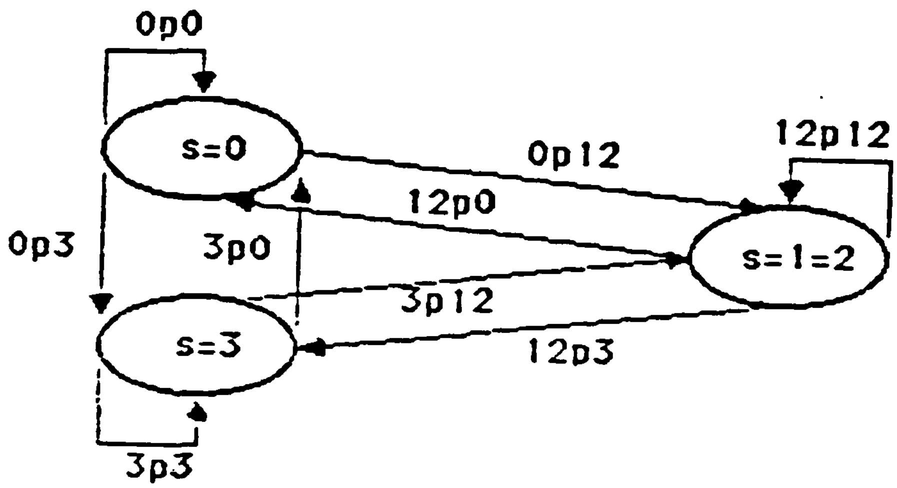


Figure 6.3 Illustration aggregated transition states of Pa.

We notice that after our first approximation and aggregation of the transition matrix P, the probabilities are very close. Indeed after certain applications of diagnostic Algorithm_1. They are exactly same as seen in following tables.

TABLE 3

THE PROBABILITY OF REPAIR OF THE

(1/2 nodes bad & 2/10 spares bad) SYSTEM

# OF STEPS	APPROX.	EXACT
1. application of algorithm 1	0.6889	0.6294
2. " "	0.8742	0.8589
3. " "	0.9492	0.9452
4. " "	0.9794	0.9784
5. " "	0.9917	0.9914
6. " "	0.9966	0.9966
7. " "	0.9986	0.9986
8. " "	0.9995	0.9994
9. " "	0.9998	0.9998
10. " "	0.9999	0.9999
11. " "	1.0000	1.0000

TABLE 4

THE PROBABILITY OF REPAIR OF THE

(2/3 nodes bad & 2/10 spares bad) SYSTEM

# OF STEPS	APPROX.	EXACT
1. application of algorithm 1	0.0138	0.1712
2. " "	0.4921	0.5205
3. " "	0.7384	0.7438
4. " "	0.8653	0.8663
5. " "	0.9306	0.9308
6. " "	0.9643	0.9643
7. " "	0.9816	0.9816
8. " "	0.9905	0.9905
9. " "	0.9951	0.9951
10. " "	0.9975	0.9975
11. " "	0.9987	0.9987
12. " "	0.9993	0.9993
13. " "	0.9997	0.9997
14. " "	0.9998	0.9998
15. " "	0.9999	0.9999
16. " "	1.0000	1.0000

TABLE 5

THE PROBABILITY OF REPAIR OF THE

(3/4 nodes bad & 2/10 spares bad) SYSTEM

# OF STEPS	APPROX.	EXACT
1. application of algorithm 1	-0.3798	0.0410
2. " "	0.1401	0.2430
3. " "	0.4641	0.4806
4. " "	0.6660	0.6624
5. " "	0.7919	0.7851
6. " "	0.8703	0.8643
7. " "	0.9192	0.9146
8. " "	0.9496	0.9463
9. " "	0.9686	0.9663
10. " "	0.9804	0.9788
11. " "	0.9878	0.9867
12. " "	0.9924	0.9917
13. " "	0.9953	0.9948
14. " "	0.9970	0.9967
15. " "	0.9982	0.9979
16. " "	0.9989	0.9987
17. " "	0.9993	0.9992
18. " "	0.9996	0.9995
19. " "	0.9997	0.9997
20. " "	0.9998	0.9998
21. " "	0.9999	0.9999
22. " "	0.9999	0.9999
23. " "	1.0000	1.0000

TABLE 6

THE PROBABILITY OF REPAIR OF THE

(4/5 nodes bad & 2/10 spares bad) SYSTEM

# OF STEPS	APPROX.	EXACT
1. application of algorithm 1	-0.5503	0.0091
2. " "	-0.1205	0.0957
3. " "	0.1901	0.2637
4. " "	0.4146	0.4384
5. " "	0.5769	0.5844
6. " "	0.6942	0.6965
7. " "	0.7789	0.7796
8. " "	0.8402	0.8404
9. " "	0.8845	0.8845
10. " "	0.9165	0.9165
11. " "	0.9397	0.9396
12. " "	0.9564	0.9564
13. " "	0.9685	0.9685
14. " "	0.9772	0.9772
15. " "	0.9835	0.9835
16. " "	0.9881	0.9881
17. " "	0.9914	0.9914
18. " "	0.9938	0.9938
19. " "	0.9955	0.9955
20. " "	0.9968	0.9967
21. " "	0.9977	0.9976
22. " "	0.9983	0.9983
23. " "	0.9988	0.9988
24. " "	0.9991	0.9991
25. " "	0.9994	0.9994
26. " "	0.9995	0.9995
27. " "	0.9997	0.9997
28. " "	0.9998	0.9998
29. " "	0.9998	0.9998
30. " "	0.9999	0.9999
31. " "	0.9999	0.9999
32. " "	0.9999	0.9999
33. " "	1.0000	1.0000

VII. CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

A. CONCLUSIONS

In this thesis, we have considered the effect of faulty spares on the probability of repair of digital systems using Algorithm_1. The analysis utilizes the graph-theoretic model of Preparata_Metze_Chien [Ref. 1] and t/s measure of Friedman [Ref. 6].

We realized that for an n node system, we need a $2^n \times 2^n$ matrix, which is very large for even moderate size n. MATLAB can not handle this size of matrix. Then, we get our first approximation which is Equation 6.3 for two node system.

In Figure 7.1, we have the probability of repair for two node system. Indeed, exact and approximate values are very close. For example, after first application of Algorithm_1, we have an exact value which is 0.6294 and an approximate value which is 0.6889. After the second application of Algorithm_1, we have an exact value which is 0.8589 and an approximate value which is 0.8742. Finally, after six applications of Algorithm_1, we have the same exact and approximate values.

In Figure 7.2, we have the probability of repair for a three node system which has two faulty nodes. After sixteen applications of Algorithm_1, the three node system is

totally fault-free. The exact and approximate values are very close also. After six applications of Algorithm_1, we have the same exact and approximate values.

In Figure 7.3, we have the probability of repair for a four node system which has three faulty nodes. In Figure 7.4, we have the probability of repair for a five node system which has four faulty nodes.

TABLE 7

OF NODES IN THE SYSTEM VS
AGGREGATED TRANSITION MATRIX DIMENSION

# OF NODES	AGGREGATED TRANSITION MATRIX DIMENSION
2	3 X 3
3	4 x 4
4	5 x 5
.	. .
.	. .
.	. .
n	(n+1) x (n+1)

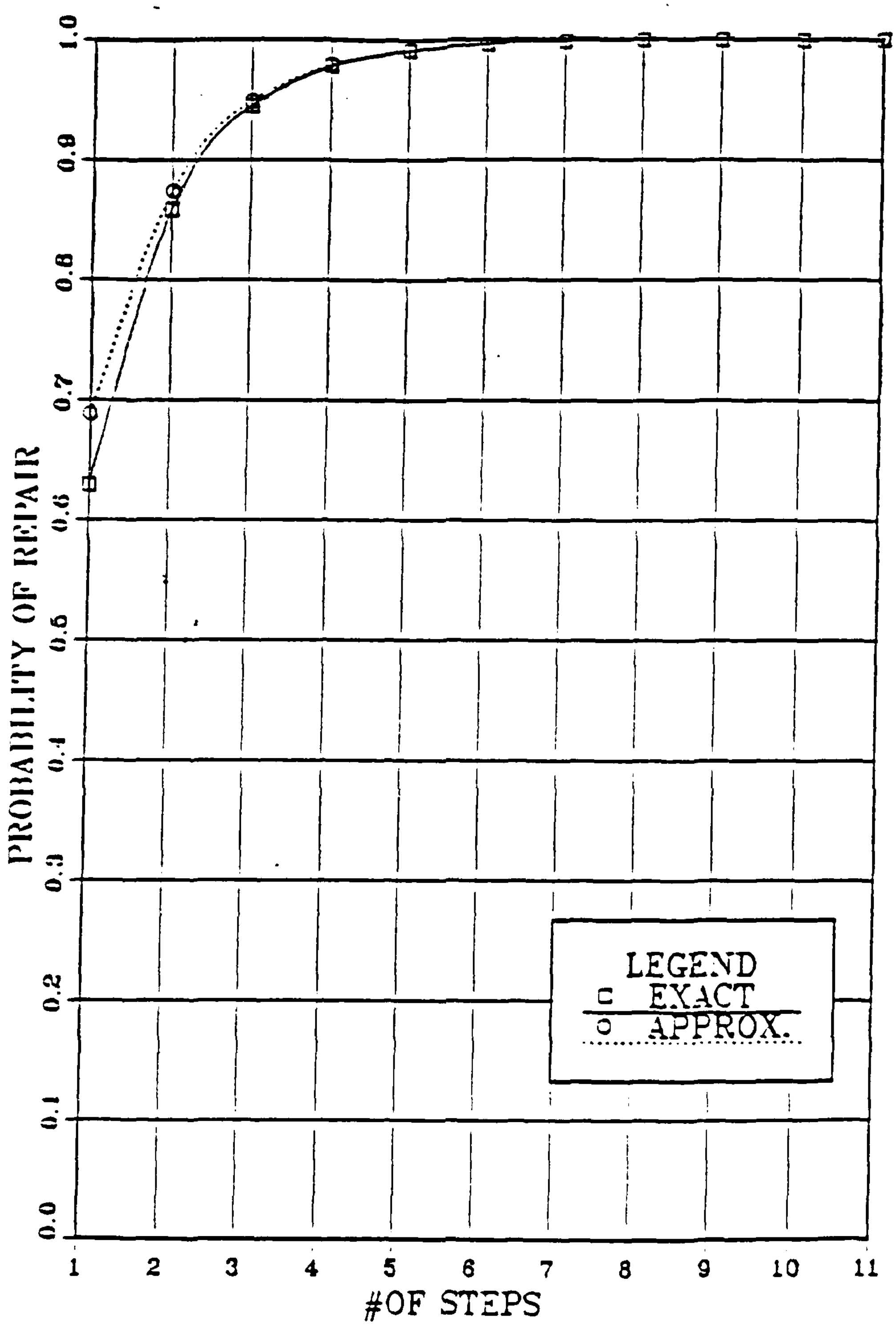


Figure 7.1 Probability of repair for (1/2 nodes bad and 2/10 spares bad) system.

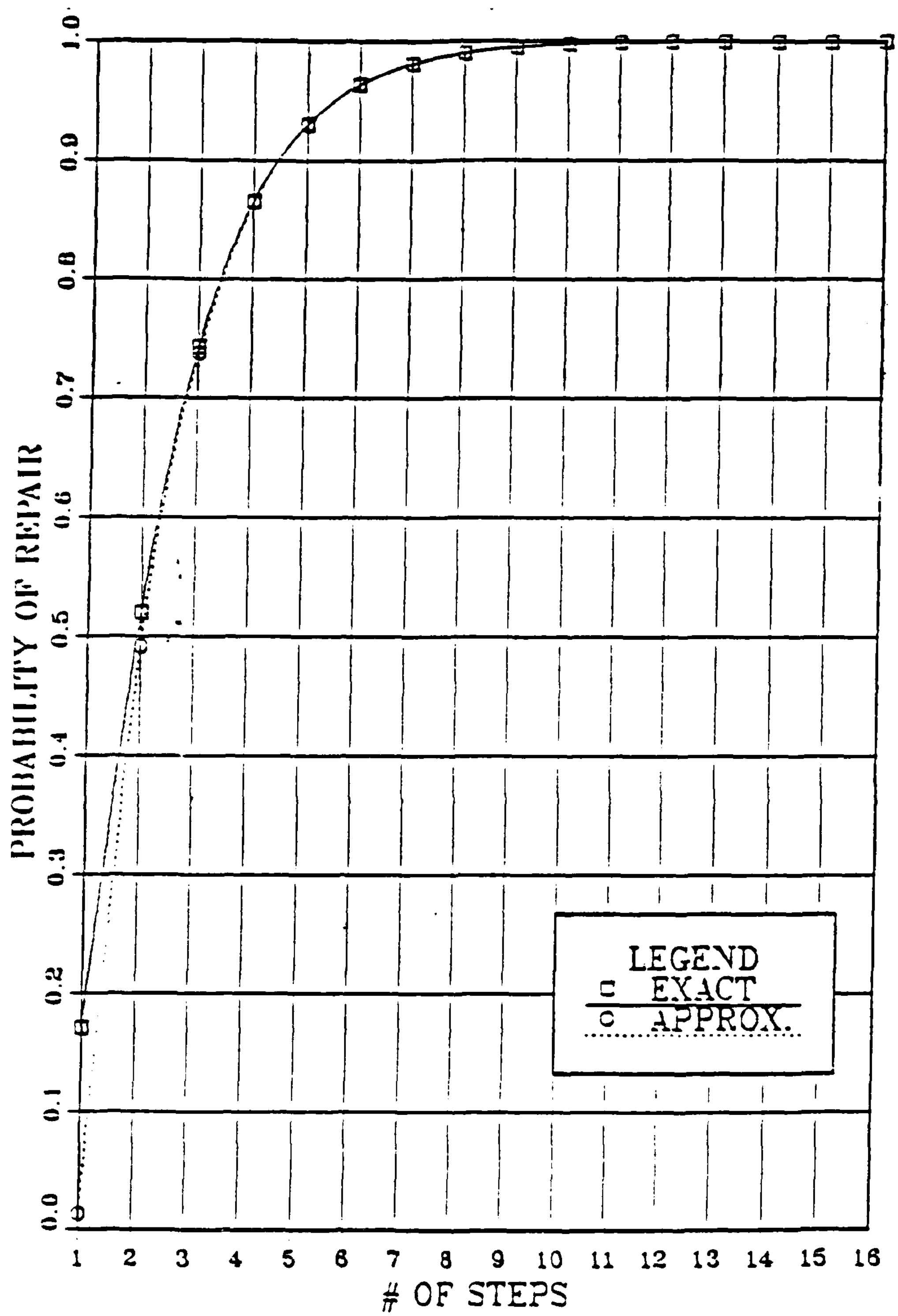


Figure 7.2 Probability of repair for (2/3 nodes bad and 2/10 spares bad) system.

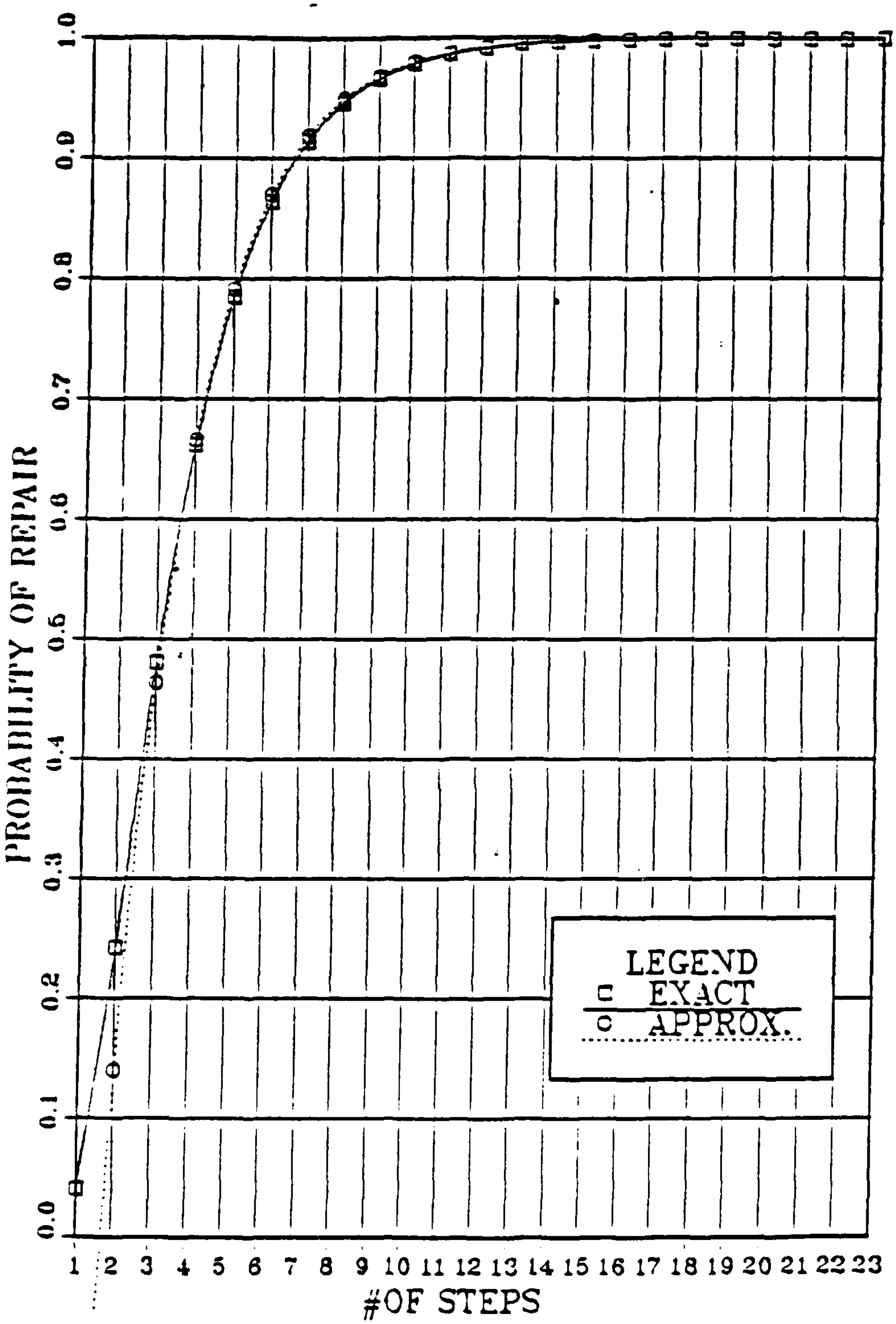


Figure 7.3 Probability of repair for (3/4 nodes bad and 2/10 spares bad) system.

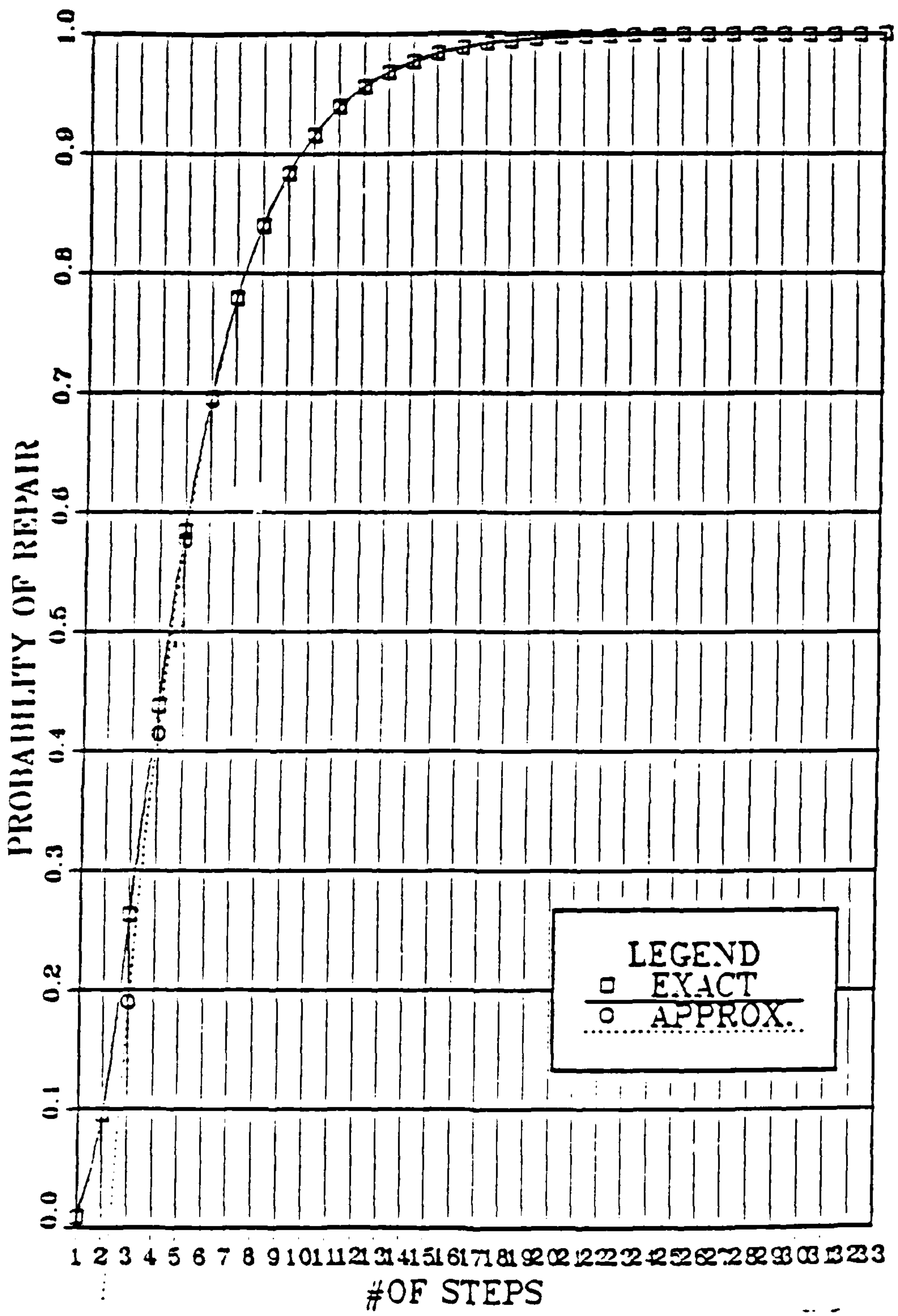


Figure 7.4 Probability of repair for (4/5 nodes bad and 2/10 spares bad) system.

After that we calculated our aggregated transition matrix Pa as a function of the elements of transition matrix P. As we see in Table 7 after aggregation, we need only matrices of dimension (n+1) x (n+1) instead of a matrix of dimension 2^n x 2^n.

We calculated the probability of repair for two, three, four and five node systems (approximate vs. exact) in Figures 7.1, 7.2, 7.3 and 7.4. After that we notice our first approximation and aggregation of transition matrix P the probabilities are almost the same. In fact after certain steps of diagnostic Algorithm_1, they are exactly the same.

B. SUGGESTIONS FOR FUTURE WORK

In this work, we defined an approach which calculates each transition state probability. In this thesis, we use Algorithm_1 to get probability of repair for two, three, four and five node system. We have very close agreements with exact solutions.

One fruitful area of research might be to extend this analysis to Algorithm_2 and 3, deriving approximation equations for the probability of repair.

So far we have done many calculations for two node systems. Three, four and five node systems need many more calculations. Especially, a computer program can be written to find the transition matrix P. It is hoped that this work shows the way for future research.

APPENDIX A

1/2 NODES BAD AND 2/10 SPARES BAD SYSTEM CASE RESULTS

This appendix contains matrix manipulations for two node system. (Faulty node=1/Faulty spares=2/Fault-free spares=8)

$$P = \begin{bmatrix} 1 & 0.6294 & 0.6294 & 0.1666 \\ 0 & 0.2312 & 0.1312 & 0.2667 \\ 0 & 0.1312 & 0.2312 & 0.2667 \\ 0 & 0.0082 & 0.0082 & 0.3000 \end{bmatrix}_{4x4}$$

4 EIGENVECTORS OF THE TRANSITION MATRIX

$$V0=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4x1} \quad V1=\begin{bmatrix} -0.0000 \\ 1.0000 \\ -1.0000 \\ -0.0000 \end{bmatrix}_{4x1} \quad V2=\begin{bmatrix} 1.0000 \\ -0.4636 \\ -0.4636 \\ -0.0729 \end{bmatrix}_{4x1} \quad V3=\begin{bmatrix} 1.0000 \\ -0.6216 \\ -0.6216 \\ 0.2431 \end{bmatrix}_{4x1}$$

4 EIGENVALUES OF	E0=1.0000
THE TRANSITION	E1=0.1000
MATRIX	E2=0.4043
	E3=0.2581

INVERSE MATRIX OF EIGENVECTORS

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 0.5000 & -0.5000 & -0.0000 \\ 0 & -0.7694 & -0.7694 & -3.9338 \\ 0 & -0.2306 & -0.2306 & 2.9338 \end{bmatrix}_{4x4}$$

1st Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.6294 & 0.6294 & 0.1666 \\ 0 & 0.2312 & 0.1312 & 0.2667 \\ 0 & 0.1312 & 0.2312 & 0.2667 \\ 0 & 0.0082 & 0.0082 & 0.3000 \end{bmatrix}_{4x4}$$

2nd Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.8589 & 0.8589 & 0.5523 \\ 0 & 0.0729 & 0.0629 & 0.1767 \\ 0 & 0.0629 & 0.0729 & 0.1767 \\ 0 & 0.0054 & 0.0054 & 0.0944 \end{bmatrix}_{4x4}$$

3rd Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9452 & 0.9452 & 0.7904 \\ 0 & 0.0265 & 0.0255 & 0.0892 \\ 0 & 0.0255 & 0.0265 & 0.0892 \\ 0 & 0.0027 & 0.0027 & 0.0312 \end{bmatrix}_{4x4}$$

4th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9784 & 0.9784 & 0.9079 \\ 0 & 0.0102 & 0.0101 & 0.0406 \\ 0 & 0.0101 & 0.0102 & 0.0406 \\ 0 & 0.0012 & 0.0012 & 0.0108 \end{bmatrix}_{4x4}$$

5th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9914 & 0.9914 & 0.9609 \\ 0 & 0.0040 & 0.0040 & 0.0176 \\ 0 & 0.0040 & 0.0040 & 0.0176 \\ 0 & 0.0005 & 0.0005 & 0.0039 \end{bmatrix}_{4x4}$$

6th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9966 & 0.9966 & 0.9837 \\ 0 & 0.0016 & 0.0016 & 0.0074 \\ 0 & 0.0016 & 0.0016 & 0.0074 \\ 0 & 0.0002 & 0.0002 & 0.0015 \end{bmatrix}_{4x4}$$

7th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9986 & 0.9986 & 0.9933 \\ 0 & 0.0006 & 0.0006 & 0.0031 \\ 0 & 0.0006 & 0.0006 & 0.0031 \\ 0 & 0.0001 & 0.0001 & 0.0006 \end{bmatrix}_{4x4}$$

8th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9994 & 0.9994 & 0.9972 \\ 0 & 0.0003 & 0.0003 & 0.0013 \\ 0 & 0.0003 & 0.0003 & 0.0013 \\ 0 & 0.0000 & 0.0000 & 0.0002 \end{bmatrix}_{4x4}$$

9th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9998 & 0.9998 & 0.9989 \\ 0 & 0.0001 & 0.0001 & 0.0005 \\ 0 & 0.0001 & 0.0001 & 0.0005 \\ 0 & 0.0000 & 0.0000 & 0.0001 \end{bmatrix}_{4x4}$$

10th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9999 & 0.9999 & 0.9995 \\ 0 & 0.0000 & 0.0000 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

11th Power of Transition matrix P

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 0.9998 \\ 0 & 0.0000 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

12th Power of Transition matrix P

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 0.9999 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

13th Power of Transition matrix P

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

Note : The (1/2 nodes bad & 2/10 spares bad) system is repaired after 13 applications of the Algorithm_1.

AGGREGATED TRANSITION MATRIX OF THE 2 NODES SYSTEM

$$Pa = \begin{bmatrix} 1 & 0.6294 & 0.1666 \\ 0 & 0.3624 & 0.5334 \\ 0 & 0.0082 & 0.3000 \end{bmatrix}_{3x3}$$

3 EIGENVECTORS OF THE AGGREGATED TRANSITION MATRIX

$$V0=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{3x1} \quad V1=\begin{bmatrix} 1.0000 \\ -0.9271 \\ -0.0729 \end{bmatrix}_{3x1} \quad V2=\begin{bmatrix} -0.8044 \\ 1.0000 \\ -0.1956 \end{bmatrix}_{3x1}$$

3 EIGENVALUES OF	E0=1.0000
THE AGGREGATED	E1=0.4043
TRANSITION MATRIX	E2=0.2581

INVERSE MATRIX OF EIGENVECTORS

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 \\ 0 & -0.7694 & -3.9338 \\ 0 & 0.2867 & -3.6472 \end{bmatrix}_{3x3}$$

1st Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.6294 & 0.1666 \\ 0 & 0.3624 & 0.5334 \\ 0 & 0.0082 & 0.3000 \end{bmatrix}_{3x3}$$

2nd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.8589 & 0.5523 \\ 0 & 0.1357 & 0.3533 \\ 0 & 0.0054 & 0.0944 \end{bmatrix}_{3x3}$$

3rd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9452 & 0.7904 \\ 0 & 0.0521 & 0.1784 \\ 0 & 0.0027 & 0.0312 \end{bmatrix}_{3x3}$$

4th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9784 & 0.9079 \\ 0 & 0.0203 & 0.0813 \\ 0 & 0.0012 & 0.0108 \end{bmatrix}_{3x3}$$

5th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9914 & 0.9609 \\ 0 & 0.0080 & 0.0352 \\ 0 & 0.0005 & 0.0039 \end{bmatrix}_{3x3}$$

6th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9966 & 0.9837 \\ 0 & 0.0032 & 0.0149 \\ 0 & 0.0002 & 0.0015 \end{bmatrix}_{3x3}$$

7th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9986 & 0.9933 \\ 0 & 0.0013 & 0.0062 \\ 0 & 0.0001 & 0.0006 \end{bmatrix}_{3x3}$$

8th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9994 & 0.9972 \\ 0 & 0.0005 & 0.0025 \\ 0 & 0.0000 & 0.0002 \end{bmatrix}_{3x3}$$

9th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9998 & 0.9989 \\ 0 & 0.0002 & 0.0010 \\ 0 & 0.0000 & 0.0001 \end{bmatrix}_{3x3}$$

10th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9995 \\ 0 & 0.0001 & 0.0004 \\ 0 & 0.0000 & 0.0000 \end{bmatrix}_{3x3}$$

11th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 0.9998 \\ 0 & 0.0000 & 0.0002 \\ 0 & 0.0000 & 0.0000 \end{bmatrix}_{3x3}$$

12th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 0.9999 \\ 0 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 \end{bmatrix}_{3x3}$$

13th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 \\ 0 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 \end{bmatrix}_{3x3}$$

Note : After aggregation the (1/2 nodes bad & 2/10 spares bad) system is also repaired after 13 applications of the Algorithm_1.

APPENDIX B

2/3 NODES BAD & 2/10 SPARES BAD SYSTEM CASE RESULTS

This appendix contains matrix manipulations for three node system. (Faulty nodes=2/Faulty spares=2/Fault-free spares=8)

$$P=\begin{bmatrix} 1 & 0.5091 & 0.5091 & 0.5091 & 0.1712 & 0.1712 & 0.1712 & 0.0548 \\ 0 & 0.3091 & 0 & 0.1591 & 0.1220 & 0.0565 & 0.4368 & 0.1068 \\ 0 & 0.1591 & 0.3091 & 0 & 0.4368 & 0.1220 & 0.0565 & 0.1068 \\ 0 & 0 & 0.1591 & 0.3091 & 0.0565 & 0.4368 & 0.1220 & 0.1068 \\ 0 & 0.0227 & 0 & 0 & 0.1277 & 0.0081 & 0.0736 & 0.1541 \\ 0 & 0 & 0.0227 & 0 & 0.0736 & 0.1277 & 0.0081 & 0.1541 \\ 0 & 0 & 0 & 0.0227 & 0.0081 & 0.0736 & 0.1277 & 0.1541 \\ 0 & 0 & 0 & 0 & 0.0041 & 0.0041 & 0.0041 & 0.1625 \end{bmatrix}_{8\times 8}$$

8 EIGENVECTORS OF THE TRANSITION MATRIX

$$V0=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad V1=\begin{bmatrix} 1.0000 \\ -0.3096 \\ -0.3096 \\ -0.3096 \\ -0.0234 \\ -0.0234 \\ -0.0234 \\ -0.0008 \end{bmatrix} \quad V2=\begin{bmatrix} 0.0000 - 0.0000i \\ -0.5000 + 0.8660i \\ 1.0000 - 0.0000i \\ -0.5000 - 0.8660i \\ 0.0158 + 0.1148i \\ 0.0916 - 0.0711i \\ -0.1073 - 0.0438i \\ -0.0000 - 0.0000i \end{bmatrix} \quad V3=\begin{bmatrix} 0.0000 + 0.0000i \\ -0.5000 - 0.8660i \\ 1.0000 + 0.0000i \\ -0.5000 + 0.8660i \\ 0.0158 - 0.1148i \\ 0.0916 + 0.0711i \\ -0.1073 + 0.0438i \\ -0.0000 + 0.0000i \end{bmatrix}$$

$$V4=\begin{bmatrix} 1.0000 \\ -0.6094 \\ -0.6094 \\ -0.6094 \\ 0.2501 \\ 0.2501 \\ 0.2501 \\ 0.0778 \end{bmatrix} \quad V5=\begin{bmatrix} 0.0000 - 0.0000i \\ -0.5000 + 0.8660i \\ 1.0000 + 0.0000i \\ -0.5000 - 0.8660i \\ -0.3786 - 0.4073i \\ -0.1634 + 0.5315i \\ 0.5420 - 0.1243i \\ 0.0000 - 0.0000i \end{bmatrix} \quad V6=\begin{bmatrix} 0.0000 + 0.0000i \\ -0.5000 - 0.8660i \\ 1.0000 - 0.0000i \\ -0.5000 + 0.8660i \\ -0.3786 + 0.4073i \\ -0.1634 - 0.5315i \\ 0.5420 + 0.1243i \\ 0.0000 + 0.0000i \end{bmatrix} \quad V7=\begin{bmatrix} 1.0000 \\ -0.7113 \\ -0.7113 \\ -0.7113 \\ 0.4218 \\ 0.4218 \\ 0.4218 \\ -0.1316 \end{bmatrix}$$

8 EIGENVALUES OF THE TRANSITION MATRIX	E0=1.0000
	E1=0.5150
	E2=0.2415 + 0.1768i
	E3=0.2415 - 0.1768i
	E4=0.2020
	E5=0.0749 + 0.0177i
	E6=0.0749 - 0.0177i
	E7=0.1231

INVERSE MATRIX OF EIGENVECTORS

1	1.0000 - 0.0000i	1.0000 + 0.0000i	1.0000 + 0.0000i	1.0000 - 0.0000i	1.0000 + 0.0000i	1.0000 - 0.0000i	1.0000
0	-0.9287 + 0.0000i	-0.9287 - 0.0000i	-0.9287 - 0.0000i	-1.9150 + 0.0000i	-1.9150 - 0.0000i	-1.9150 + 0.0000i	-3.3556
0	-0.1659 - 0.2296i	0.2818 - 0.0288i	-0.1159 + 0.2585i	0.3070 - 0.4065i	0.1985 + 0.4691i	-0.5055 - 0.0627i	-0.0000
0	-0.1659 + 0.2296i	0.2818 + 0.0288i	-0.1159 - 0.2585i	0.3070 + 0.4065i	0.1985 - 0.4691i	-0.5055 + 0.0627i	-0.0000
0	-0.0484 + 0.0000i	-0.0484 - 0.0000i	-0.0484 - 0.0000i	0.5674 - 0.0000i	0.5674 - 0.0000i	0.5674 + 0.0000i	6.2433
0	-0.0008 - 0.0590i	0.0515 + 0.0288i	-0.0507 + 0.0302i	-0.3070 + 0.4065i	-0.1985 - 0.4691i	0.5055 + 0.0627i	-0.0000
0	-0.0008 + 0.0590i	0.0515 - 0.0288i	-0.0507 - 0.0302i	-0.3070 - 0.4065i	-0.1985 + 0.4691i	0.5055 - 0.0627i	-0.0000
0	-0.0229 + 0.0000i	-0.0229 - 0.0000i	-0.0229 - 0.0000i	0.3475 - 0.0000i	0.3475 + 0.0000i	0.3475 + 0.0000i	-3.8878

1st Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.5091 & 0.5091 & 0.5091 & 0.1712 & 0.1712 & 0.1712 & 0.0548 \\ 0 & 0.3091 & 0 & 0.1591 & 0.1220 & 0.0565 & 0.4368 & 0.1068 \\ 0 & 0.1591 & 0.3091 & 0 & 0.4368 & 0.1220 & 0.0565 & 0.1068 \\ 0 & 0 & 0.1591 & 0.3091 & 0.0565 & 0.4368 & 0.1220 & 0.1068 \\ 0 & 0.0227 & 0 & 0 & 0.1277 & 0.0081 & 0.0736 & 0.1541 \\ 0 & 0 & 0.0227 & 0 & 0.0736 & 0.1277 & 0.0081 & 0.1541 \\ 0 & 0 & 0 & 0.0227 & 0.0081 & 0.0736 & 0.1277 & 0.1541 \\ 0 & 0 & 0 & 0 & 0.0041 & 0.0041 & 0.0041 & 0.1625 \end{bmatrix}_{8x8}$$

2nd Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.7513 & 0.7513 & 0.7513 & 0.5205 & 0.5205 & 0.5205 & 0.3060 \\ 0 & 0.0983 & 0.0266 & 0.1083 & 0.0704 & 0.1277 & 0.2201 & 0.1622 \\ 0 & 0.1083 & 0.0983 & 0.0266 & 0.2201 & 0.0704 & 0.1277 & 0.1622 \\ 0 & 0.0266 & 0.1083 & 0.0983 & 0.1277 & 0.2201 & 0.0704 & 0.1622 \\ 0 & 0.0099 & 0.0002 & 0.0053 & 0.0209 & 0.0094 & 0.0294 & 0.0597 \\ 0 & 0.0053 & 0.0099 & 0.0002 & 0.0294 & 0.0209 & 0.0094 & 0.0597 \\ 0 & 0.0002 & 0.0053 & 0.0099 & 0.0094 & 0.0294 & 0.0209 & 0.0597 \\ 0 & 0.0001 & 0.0001 & 0.0001 & 0.0015 & 0.0015 & 0.0015 & 0.0283 \end{bmatrix}_{8x8}$$

3rd Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.8727 & 0.8727 & 0.8727 & 0.7438 & 0.7438 & 0.7438 & 0.5859 \\ 0 & 0.0362 & 0.0283 & 0.0541 & 0.0506 & 0.0898 & 0.0926 & 0.1157 \\ 0 & 0.0541 & 0.0362 & 0.0283 & 0.0926 & 0.0506 & 0.0898 & 0.1157 \\ 0 & 0.0283 & 0.0541 & 0.0362 & 0.0898 & 0.0926 & 0.0506 & 0.1157 \\ 0 & 0.0036 & 0.0011 & 0.0039 & 0.0054 & 0.0067 & 0.0106 & 0.0206 \\ 0 & 0.0039 & 0.0036 & 0.0011 & 0.0106 & 0.0054 & 0.0067 & 0.0206 \\ 0 & 0.0011 & 0.0039 & 0.0036 & 0.0067 & 0.0106 & 0.0054 & 0.0206 \\ 0 & 0.0001 & 0.0001 & 0.0001 & 0.0005 & 0.0005 & 0.0005 & 0.0053 \end{bmatrix}_{8x8}$$

4th Power of Transition matrix P

$$\begin{bmatrix} 1 & 0.9346 & 0.9346 & 0.9346 & 0.8663 & 0.8663 & 0.8663 & 0.7735 \\ 0 & 0.0169 & 0.0194 & 0.0246 & 0.0342 & 0.0483 & 0.0408 & 0.0674 \\ 0 & 0.0246 & 0.0169 & 0.0194 & 0.0408 & 0.0342 & 0.0483 & 0.0674 \\ 0 & 0.0194 & 0.0246 & 0.0169 & 0.0483 & 0.0408 & 0.0342 & 0.0674 \\ 0 & 0.0014 & 0.0011 & 0.0020 & 0.0025 & 0.0038 & 0.0040 & 0.0078 \\ 0 & 0.0020 & 0.0014 & 0.0011 & 0.0040 & 0.0025 & 0.0038 & 0.0078 \\ 0 & 0.0011 & 0.0020 & 0.0014 & 0.0038 & 0.0040 & 0.0025 & 0.0078 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0002 & 0.0002 & 0.0002 & 0.0011 \end{bmatrix}_{8x8}$$

5th Power of Transition matrix P

$$
\begin{bmatrix}
1 & 0.9663 & 0.9663 & 0.9663 & 0.9308 & 0.9308 & 0.9308 & 0.8804 \\
0 & 0.0091 & 0.0110 & 0.0112 & 0.0204 & 0.0238 & 0.0198 & 0.0364 \\
0 & 0.0112 & 0.0091 & 0.0110 & 0.0198 & 0.0204 & 0.0238 & 0.0364 \\
0 & 0.0110 & 0.0112 & 0.0091 & 0.0238 & 0.0198 & 0.0204 & 0.0364 \\
0 & 0.0007 & 0.0007 & 0.0009 & 0.0014 & 0.0019 & 0.0017 & 0.0033 \\
0 & 0.0009 & 0.0007 & 0.0007 & 0.0017 & 0.0014 & 0.0019 & 0.0033 \\
0 & 0.0007 & 0.0009 & 0.0007 & 0.0019 & 0.0017 & 0.0014 & 0.0033 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0003
\end{bmatrix}_{8x8}
$$

6th Power of Transition matrix P

$$
\begin{bmatrix}
1 & 0.9827 & 0.9827 & 0.9827 & 0.9643 & 0.9643 & 0.9643 & 0.9378 \\
0 & 0.0050 & 0.0057 & 0.0054 & 0.0112 & 0.0116 & 0.0103 & 0.0191 \\
0 & 0.0054 & 0.0050 & 0.0057 & 0.0103 & 0.0112 & 0.0116 & 0.0191 \\
0 & 0.0057 & 0.0054 & 0.0050 & 0.0116 & 0.0103 & 0.0112 & 0.0191 \\
0 & 0.0004 & 0.0004 & 0.0004 & 0.0008 & 0.0009 & 0.0008 & 0.0016 \\
0 & 0.0004 & 0.0004 & 0.0004 & 0.0008 & 0.0008 & 0.0009 & 0.0016 \\
0 & 0.0004 & 0.0004 & 0.0004 & 0.0009 & 0.0008 & 0.0008 & 0.0016 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001
\end{bmatrix}_{8x8}
$$

7th Power of Transition matrix P

$$
\begin{bmatrix}
1 & 0.9911 & 0.9911 & 0.9911 & 0.9816 & 0.9816 & 0.9816 & 0.9678 \\
0 & 0.0027 & 0.0029 & 0.0027 & 0.0059 & 0.0057 & 0.0055 & 0.0099 \\
0 & 0.0027 & 0.0027 & 0.0029 & 0.0055 & 0.0059 & 0.0057 & 0.0099 \\
0 & 0.0029 & 0.0027 & 0.0027 & 0.0057 & 0.0055 & 0.0059 & 0.0099 \\
0 & 0.0002 & 0.0002 & 0.0002 & 0.0004 & 0.0005 & 0.0004 & 0.0008 \\
0 & 0.0002 & 0.0002 & 0.0002 & 0.0004 & 0.0004 & 0.0005 & 0.0008 \\
0 & 0.0002 & 0.0002 & 0.0002 & 0.0005 & 0.0004 & 0.0004 & 0.0008 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}
$$

8th Power of Transition matrix P

$$
\begin{bmatrix}
1 & 0.9954 & 0.9954 & 0.9954 & 0.9905 & 0.9905 & 0.9905 & 0.9834 \\
0 & 0.0014 & 0.0014 & 0.0014 & 0.0030 & 0.0029 & 0.0029 & 0.0051 \\
0 & 0.0014 & 0.0014 & 0.0014 & 0.0029 & 0.0030 & 0.0029 & 0.0051 \\
0 & 0.0014 & 0.0014 & 0.0014 & 0.0029 & 0.0029 & 0.0030 & 0.0051 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0004 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0004 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0004 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}
$$

9th Power of Transition matrix P

$$\begin{bmatrix}
1 & 0.9976 & 0.9976 & 0.9976 & 0.9951 & 0.9951 & 0.9951 & 0.9915 \\
0 & 0.0007 & 0.0007 & 0.0007 & 0.0015 & 0.0015 & 0.0015 & 0.0026 \\
0 & 0.0007 & 0.0007 & 0.0007 & 0.0015 & 0.0015 & 0.0015 & 0.0026 \\
0 & 0.0007 & 0.0007 & 0.0007 & 0.0015 & 0.0015 & 0.0015 & 0.0026 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

10th Power of Transition matrix P

$$\begin{bmatrix}
1 & 0.9988 & 0.9988 & 0.9988 & 0.9975 & 0.9975 & 0.9975 & 0.9956 \\
0 & 0.0004 & 0.0004 & 0.0004 & 0.0008 & 0.0008 & 0.0008 & 0.0014 \\
0 & 0.0004 & 0.0004 & 0.0004 & 0.0008 & 0.0008 & 0.0008 & 0.0014 \\
0 & 0.0004 & 0.0004 & 0.0004 & 0.0008 & 0.0008 & 0.0008 & 0.0014 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

11th Power of Transition matrix P

$$\begin{bmatrix}
1 & 0.9994 & 0.9994 & 0.9994 & 0.9987 & 0.9987 & 0.9987 & 0.9977 \\
0 & 0.0002 & 0.0002 & 0.0002 & 0.0004 & 0.0004 & 0.0004 & 0.0007 \\
0 & 0.0002 & 0.0002 & 0.0002 & 0.0004 & 0.0004 & 0.0004 & 0.0007 \\
0 & 0.0002 & 0.0002 & 0.0002 & 0.0004 & 0.0004 & 0.0004 & 0.0007 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

12th Power of Transition matrix P

$$\begin{bmatrix}
1 & 0.9997 & 0.9997 & 0.9997 & 0.9993 & 0.9993 & 0.9993 & 0.9988 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0004 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0004 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0004 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

13th Power of Transition matrix P

$$\begin{bmatrix}
1 & 0.9998 & 0.9998 & 0.9998 & 0.9997 & 0.9997 & 0.9997 & 0.9994 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\
0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

14th Power of Transition matrix P

$$\begin{bmatrix}
1 & 0.9999 & 0.9999 & 0.9999 & 0.9998 & 0.9998 & 0.9998 & 0.9997 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

15th Power of Transition matrix P

$$\begin{bmatrix}
1 & 1.0000 & 1.0000 & 1.0000 & 0.9999 & 0.9999 & 0.9999 & 0.9998 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

16th Power of Transition matrix P

$$\begin{bmatrix}
1 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.9999 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}$$

17th Power of Transition matrix P

$$
\begin{bmatrix}
1 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}_{8x8}
$$

Note : The (2/3 nodes bad & 2/10 spares bad) system is repaired after 17 applications of the Algorithm_1.

AGGREGATED TRANSITION MATRIX OF THE 3 NODES SYSTEM

$$Pa=\begin{bmatrix} 1 & 0.5091 & 0.1712 & 0.0548 \\ 0 & 0.4682 & 0.6153 & 0.3204 \\ 0 & 0.0227 & 0.2094 & 0.4623 \\ 0 & 0 & 0.0041 & 0.1625 \end{bmatrix}_{4x4}$$

4 EIGENVECTORS OF THE AGGREGATED TRANSITION MATRIX

$$V0=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4x1} \quad V1=\begin{bmatrix} 1.0000 \\ -0.9289 \\ -0.0702 \\ -0.0008 \end{bmatrix}_{4x1} \quad V2=\begin{bmatrix} -0.5470 \\ 1.0000 \\ -0.4104 \\ -0.0426 \end{bmatrix}_{4x1} \quad V3=\begin{bmatrix} -0.4686 \\ 1.0000 \\ -0.5930 \\ 0.0617 \end{bmatrix}_{4x1}$$

4 EIGENVALUES OF	E0=1.0000
THE AGGREGATED	E1=0.5150
TRANSITION MATRIX	E2=0.2020
	E3=0.1231

INVERSE MATRIX OF EIGENVECTORS

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 \\ 0 & -0.9287 & -1.9150 & -3.3556 \\ 0 & 0.0885 & -1.0373 & -11.4133 \\ 0 & 0.0488 & -0.7415 & 8.2962 \end{bmatrix}_{4x4}$$

1st Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.5091 & 0.1712 & 0.0548 \\ 0 & 0.4682 & 0.6153 & 0.3204 \\ 0 & 0.0227 & 0.2094 & 0.4623 \\ 0 & 0 & 0.0041 & 0.1625 \end{bmatrix}_{4x4}$$

2nd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.7513 & 0.5205 & 0.3060 \\ 0 & 0.2332 & 0.4182 & 0.4865 \\ 0 & 0.0154 & 0.0597 & 0.1792 \\ 0 & 0.0001 & 0.0015 & 0.0283 \end{bmatrix}_{4x4}$$

3rd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.8727 & 0.7438 & 0.5859 \\ 0 & 0.1187 & 0.2330 & 0.3471 \\ 0 & 0.0086 & 0.0227 & 0.0617 \\ 0 & 0.0001 & 0.0005 & 0.0053 \end{bmatrix}_{4x4}$$

4th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9346 & 0.8663 & 0.7735 \\ 0 & 0.0609 & 0.1232 & 0.2022 \\ 0 & 0.0045 & 0.0103 & 0.0233 \\ 0 & 0.0000 & 0.0002 & 0.0011 \end{bmatrix}_{4x4}$$

5th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9663 & 0.9308 & 0.8804 \\ 0 & 0.0313 & 0.0641 & 0.1093 \\ 0 & 0.0024 & 0.0050 & 0.0100 \\ 0 & 0.0000 & 0.0001 & 0.0003 \end{bmatrix}_{4x4}$$

6th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9827 & 0.9643 & 0.9378 \\ 0 & 0.0161 & 0.0331 & 0.0574 \\ 0 & 0.0012 & 0.0025 & 0.0047 \\ 0 & 0.0000 & 0.0000 & 0.0001 \end{bmatrix}_{4x4}$$

7th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9911 & 0.9816 & 0.9678 \\ 0 & 0.0083 & 0.0171 & 0.0298 \\ 0 & 0.0006 & 0.0013 & 0.0023 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

8th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9954 & 0.9905 & 0.9834 \\ 0 & 0.0043 & 0.0088 & 0.0154 \\ 0 & 0.0003 & 0.0007 & 0.0012 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

9th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9976 & 0.9951 & 0.9915 \\ 0 & 0.0022 & 0.0045 & 0.0079 \\ 0 & 0.0002 & 0.0003 & 0.0006 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

10th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9988 & 0.9975 & 0.9956 \\ 0 & 0.0011 & 0.0023 & 0.0041 \\ 0 & 0.0001 & 0.0002 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

11th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9994 & 0.9987 & 0.9977 \\ 0 & 0.0006 & 0.0012 & 0.0021 \\ 0 & 0.0000 & 0.0001 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

12th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9997 & 0.9993 & 0.9988 \\ 0 & 0.0003 & 0.0006 & 0.0011 \\ 0 & 0.0000 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

13th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9998 & 0.9997 & 0.9994 \\ 0 & 0.0002 & 0.0003 & 0.0006 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

14th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9998 & 0.9997 \\ 0 & 0.0001 & 0.0002 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

15th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 0.9999 & 0.9998 \\ 0 & 0.0000 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

16th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 0.9999 \\ 0 & 0.0000 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

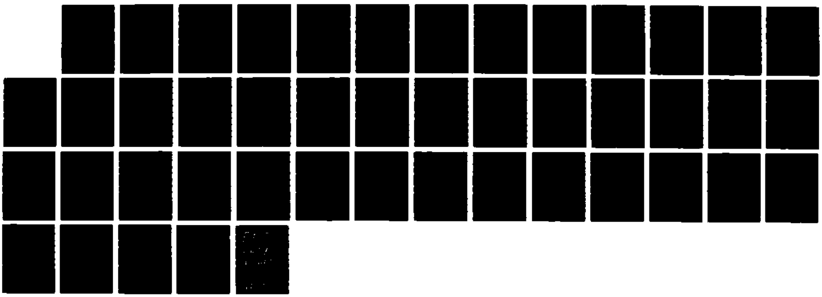
17th Power of the aggregated transition matrix Pa

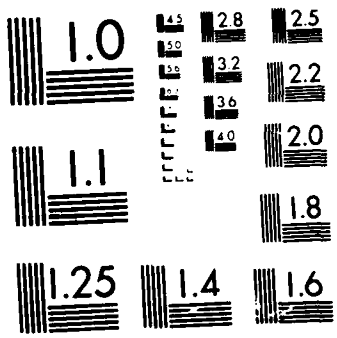
$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{4x4}$$

Note : After aggregation the (2/3 nodes bad & 2/10 spares bad) system is also repaired after 17 applications of the Algorithm_1.

AD-A193 526 ANALYSIS OF THE EFFECT OF FAULTY SPARES ON THE PERFORMANCE OF DIAGNOSTIC ALGORITHMS IN RELIABLE SYSTEMS(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA 2/2

UNCLASSIFIED M PAKTUNA DEC 87 F/G 12/6 NL




1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6

APPENDIX C

3/4 NODES BAD & 2/10 SPARES BAD SYSTEM CASE RESULTS

This appendix contains matrix manipulations for four node system. (Faulty nodes=3/Faulty spares=2/Fault-free spares=8)

States 0 through 6

P=

1.0000	0.4087	0.4087	0.4087	0.4087	0.1099	0.1099
0	0.3739	0	0	0.1739	0.3888	0
0	0.1739	0.3739	0	0	0.0547	0.1342
0	0	0.1739	0.3739	0	0	0.3888
0	0	0	0.1739	0.3739	0.1342	0.0547
0	0	0	0	0.0435	0.1842	0
0	0	0.0435	0	0	0	0.1842
0	0	0	0	0	0.0182	0.0182
0	0	0	0	0	0	0
0	0.0435	0	0	0	0.0978	0
0	0	0	0.0435	0	0	0.0978
0	0	0	0	0	0	0.0122
0	0	0	0	0	0	0
0	0	0	0	0	0.0122	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

States 7 through 13

0.1505	0.1505	0.1099	0.1099	0.0410	0.0410	0.0410
0.0852	0.2194	0.1342	0.0547	0.0204	0.1342	0.1342
0.2194	0.0852	0.3888	0	0.0486	0.0204	0.1342
0.0852	0.2194	0.0547	0.1342	0.1342	0.0486	0.0204
0.2194	0.0852	0	0.3888	0.1342	0.1342	0.0486
0.0365	0.0365	0	0.0978	0.0333	0.2885	0.0702
0.0365	0.0365	0.0978	0	0.0702	0.0051	0.0333
0.0662	0.0183	0	0	0.0702	0.0333	0.0702
0.0183	0.0662	0.0182	0.0182	0.0333	0.0702	0.0333
0.0365	0.0365	0.1842	0	0.0051	0.0333	0.2885
0.0365	0.0365	0	0.1842	0.2885	0.0702	0.0051
0.0029	0.0020	0	0	0.0699	0.0041	0.0041
0.0020	0.0029	0	0.0122	0.0409	0.0699	0.0041
0.0029	0.0020	0	0	0.0041	0.0409	0.0699
0.0020	0.0029	0.0122	0	0.0041	0.0041	0.0409
0	0	0	0	0.0020	0.0020	0.0020

States 14 through 15

0.0410	0.0171
0.0486	0.0387
0.1342	0.0387
0.1342	0.0387
0.0204	0.0387
0.0051	0.0651
0.2885	0.0651
0.0333	0.0651
0.0702	0.0651
0.0702	0.0651
0.0333	0.0651
0.0409	0.0875
0.0041	0.0875
0.0041	0.0875
0.0699	0.0875
0.0020	0.0875

16x16

1st Power of Transition matrix P

States 0 through 6

1.0000	0.4087	0.4087	0.4087	0.4087	0.1099	0.1099
0	0.3739	0	0	0.1739	0.3888	0
0	0.1739	0.3739	0	0	0.0547	0.1342
0	0	0.1739	0.3739	0	0	0.3888
0	0	0	0.1739	0.3739	0.1342	0.0547
0	0	0	0	0.0435	0.1842	0
0	0	0.0435	0	0	0	0.1842
0	0	0	0	0	0.0182	0.0182
0	0	0	0	0	0	0
0	0.0435	0	0	0	0.0978	0
0	0	0	0.0435	0	0	0.0978
0	0	0	0	0	0	0.0122
0	0	0	0	0	0	0
0	0	0	0	0	0.0122	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

States 7 through 13

0.1505	0.1505	0.1099	0.1099	0.0410	0.0410	0.0410
0.0852	0.2194	0.1342	0.0547	0.0204	0.1342	0.1342
0.2194	0.0852	0.3888	0	0.0486	0.0204	0.1342
0.0852	0.2194	0.0547	0.1342	0.1342	0.0486	0.0204
0.2194	0.0852	0	0.3888	0.1342	0.1342	0.0486
0.0365	0.0365	0	0.0978	0.0333	0.2885	0.0702
0.0365	0.0365	0.0978	0	0.0702	0.0051	0.0333
0.0662	0.0183	0	0	0.0702	0.0333	0.0702
0.0183	0.0662	0.0182	0.0182	0.0333	0.0702	0.0333
0.0365	0.0365	0.1842	0	0.0051	0.0333	0.2885
0.0365	0.0365	0	0.1842	0.2885	0.0702	0.0051
0.0029	0.0020	0	0	0.0699	0.0041	0.0041
0.0020	0.0029	0	0.0122	0.0409	0.0699	0.0041
0.0029	0.0020	0	0	0.0041	0.0409	0.0699
0.0020	0.0029	0.0122	0	0.0041	0.0041	0.0409
0	0	0	0	0.0020	0.0020	0.0020

States 14 through 15

0.0410	0.0171
0.0486	0.0387
0.1342	0.0387
0.1342	0.0387
0.0204	0.0387
0.0051	0.0651
0.2885	0.0651
0.0333	0.0651
0.0702	0.0651
0.0702	0.0651
0.0333	0.0651
0.0409	0.0875
0.0041	0.0875
0.0041	0.0875
0.0699	0.0875
0.0020	0.0875

16x16

2nd Power of Transition matrix P

States 0 through 6

1.0000	0.6374	0.6374	0.6374	0.6374	0.3802	0.3802
0	0.1456	0	0.0326	0.1470	0.2566	0.0167
0	0.1470	0.1456	0	0.0326	0.1418	0.0795
0	0.0326	0.1470	0.1456	0	0.0167	0.2566
0	0	0.0326	0.1470	0.1456	0.0795	0.1418
0	0	0	0.0118	0.0243	0.0413	0.0130
0	0.0118	0.0243	0	0	0.0130	0.0413
0	0	0.0008	0	0.0008	0.0054	0.0054
0	0.0008	0	0.0008	0	0.0025	0.0025
0	0.0243	0	0	0.0118	0.0571	0.0007
0	0	0.0118	0.0243	0	0.0007	0.0571
0	0	0.0005	0	0	0.0001	0.0032
0	0	0	0.0005	0	0.0001	0.0017
0	0	0	0	0.0005	0.0032	0.0001
0	0.0005	0	0	0	0.0017	0.0001
0	0	0	0	0	0.0000	0.0000

States 7 through 13

0.4286	0.4286	0.3802	0.3802	0.2430	0.2430	0.2430
0.1016	0.1349	0.0795	0.1418	0.0814	0.2275	0.1503
0.1349	0.1016	0.2566	0.0167	0.0586	0.0814	0.2275
0.1016	0.1349	0.1418	0.0795	0.1503	0.0586	0.0814
0.1349	0.1016	0.0167	0.2566	0.2275	0.1503	0.0586
0.0238	0.0181	0.0007	0.0571	0.0585	0.0929	0.0259
0.0238	0.0181	0.0571	0.0007	0.0259	0.0122	0.0585
0.0066	0.0042	0.0025	0.0025	0.0140	0.0146	0.0140
0.0042	0.0066	0.0054	0.0054	0.0146	0.0140	0.0146
0.0181	0.0238	0.0413	0.0130	0.0122	0.0585	0.0929
0.0181	0.0238	0.0130	0.0413	0.0929	0.0259	0.0122
0.0010	0.0009	0.0017	0.0001	0.0065	0.0014	0.0031
0.0009	0.0010	0.0001	0.0032	0.0097	0.0065	0.0014
0.0010	0.0009	0.0001	0.0017	0.0031	0.0097	0.0065
0.0009	0.0010	0.0032	0.0001	0.0014	0.0031	0.0097
0.0000	0.0000	0.0000	0.0000	0.0004	0.0004	0.0004

States 14 through 15

0.2430	0.1444
0.0586	0.1115
0.1503	0.1115
0.2275	0.1115
0.0814	0.1115
0.0122	0.0652
0.0929	0.0652
0.0146	0.0317
0.0140	0.0317
0.0259	0.0652
0.0585	0.0652
0.0097	0.0192
0.0031	0.0192
0.0014	0.0192
0.0065	0.0192
0.0004	0.0084

16x16

3rd Power of Transition matrix P.

States 0 through 6

1.0000	0.7744	0.7744	0.7744	0.7744	0.5961	0.5961
0	0.0579	0.0064	0.0439	0.0914	0.1351	0.0405
0	0.0914	0.0579	0.0064	0.0439	0.1259	0.0408
0	0.0439	0.0914	0.0579	0.0064	0.0405	0.1351
0	0.0064	0.0439	0.0914	0.0579	0.0408	0.1259
0	0.0000	0.0026	0.0111	0.0109	0.0117	0.0151
0	0.0111	0.0109	0.0000	0.0026	0.0151	0.0117
0	0.0002	0.0005	0.0002	0.0005	0.0017	0.0017
0	0.0005	0.0002	0.0005	0.0002	0.0016	0.0016
0	0.0109	0.0000	0.0026	0.0111	0.0270	0.0025
0	0.0026	0.0111	0.0109	0.0000	0.0025	0.0270
0	0.0002	0.0003	0.0000	0.0000	0.0003	0.0008
0	0.0000	0.0002	0.0003	0.0000	0.0001	0.0010
0	0.0000	0.0000	0.0002	0.0003	0.0008	0.0003
0	0.0003	0.0000	0.0000	0.0002	0.0010	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.6329	0.6329	0.5961	0.5961	0.4806	0.4806	0.4806
0.0759	0.0818	0.0408	0.1259	0.1058	0.1633	0.0956
0.0818	0.0759	0.1351	0.0405	0.0529	0.1058	0.1633
0.0759	0.0818	0.1259	0.0408	0.0956	0.0529	0.1058
0.0818	0.0759	0.0405	0.1351	0.1633	0.0956	0.0529
0.0128	0.0109	0.0025	0.0270	0.0341	0.0299	0.0106
0.0128	0.0109	0.0270	0.0025	0.0106	0.0139	0.0341
0.0016	0.0013	0.0016	0.0016	0.0038	0.0043	0.0038
0.0013	0.0016	0.0017	0.0017	0.0043	0.0038	0.0043
0.0109	0.0128	0.0117	0.0151	0.0139	0.0341	0.0299
0.0109	0.0128	0.0151	0.0117	0.0299	0.0106	0.0139
0.0004	0.0004	0.0010	0.0001	0.0010	0.0005	0.0015
0.0004	0.0004	0.0003	0.0008	0.0022	0.0010	0.0005
0.0004	0.0004	0.0001	0.0010	0.0015	0.0022	0.0010
0.0004	0.0004	0.0008	0.0003	0.0005	0.0015	0.0022
0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001

States 14 through 15

0.4806	0.3683
0.0529	0.1152
0.0956	0.1152
0.1633	0.1152
0.1058	0.1152
0.0139	0.0337
0.0299	0.0337
0.0043	0.0096
0.0038	0.0096
0.0106	0.0337
0.0341	0.0337
0.0022	0.0040
0.0015	0.0040
0.0005	0.0040
0.0010	0.0040
0.0001	0.0009

16x16

4th Power of Transition matrix P

States 0 through 6

1.0000	0.8588	0.8588	0.8588	0.8588	0.7427	0.7427
0	0.0245	0.0118	0.0378	0.0501	0.0666	0.0454
0	0.0501	0.0245	0.0118	0.0378	0.0845	0.0263
0	0.0378	0.0501	0.0245	0.0118	0.0454	0.0666
0	0.0118	0.0378	0.0501	0.0245	0.0263	0.0845
0	0.0006	0.0036	0.0072	0.0046	0.0044	0.0113
0	0.0072	0.0046	0.0006	0.0036	0.0113	0.0044
0	0.0003	0.0003	0.0003	0.0003	0.0007	0.0007
0	0.0003	0.0003	0.0003	0.0003	0.0008	0.0008
0	0.0046	0.0006	0.0036	0.0072	0.0124	0.0039
0	0.0036	0.0072	0.0046	0.0006	0.0039	0.0124
0	0.0002	0.0002	0.0000	0.0000	0.0003	0.0002
0	0.0000	0.0002	0.0002	0.0000	0.0001	0.0004
0	0.0000	0.0000	0.0002	0.0002	0.0002	0.0003
0	0.0002	0.0000	0.0000	0.0002	0.0004	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.7675	0.7675	0.7427	0.7427	0.6624	0.6624	0.6624
0.0502	0.0510	0.0263	0.0845	0.0865	0.0962	0.0554
0.0510	0.0502	0.0666	0.0454	0.0484	0.0865	0.0962
0.0502	0.0510	0.0845	0.0263	0.0554	0.0484	0.0865
0.0510	0.0502	0.0454	0.0666	0.0962	0.0554	0.0484
0.0072	0.0068	0.0039	0.0124	0.0174	0.0115	0.0062
0.0072	0.0068	0.0124	0.0039	0.0062	0.0113	0.0174
0.0007	0.0006	0.0008	0.0008	0.0014	0.0014	0.0014
0.0006	0.0007	0.0007	0.0007	0.0014	0.0014	0.0014
0.0068	0.0072	0.0044	0.0113	0.0113	0.0174	0.0115
0.0068	0.0072	0.0113	0.0044	0.0115	0.0062	0.0113
0.0002	0.0002	0.0004	0.0001	0.0003	0.0003	0.0006
0.0002	0.0002	0.0003	0.0002	0.0006	0.0003	0.0003
0.0002	0.0002	0.0001	0.0004	0.0006	0.0006	0.0003
0.0002	0.0002	0.0002	0.0003	0.0003	0.0006	0.0006
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.6624	0.5750
0.0484	0.0869
0.0554	0.0869
0.0962	0.0869
0.0865	0.0869
0.0113	0.0169
0.0115	0.0169
0.0014	0.0029
0.0014	0.0029
0.0062	0.0169
0.0174	0.0169
0.0006	0.0010
0.0006	0.0010
0.0003	0.0010
0.0003	0.0010
0.0000	0.0001

16x16

5th Power of Transition matrix P

States 0 through 6

1.0000	0.9115	0.9115	0.9115	0.9115	0.8376	0.8376
0	0.0124	0.0130	0.0265	0.0259	0.0333	0.0376
0	0.0259	0.0124	0.0130	0.0265	0.0501	0.0207
0	0.0265	0.0259	0.0124	0.0130	0.0376	0.0333
0	0.0130	0.0265	0.0259	0.0124	0.0207	0.0501
0	0.0010	0.0031	0.0040	0.0020	0.0024	0.0072
0	0.0040	0.0020	0.0010	0.0031	0.0072	0.0024
0	0.0002	0.0002	0.0002	0.0002	0.0004	0.0004
0	0.0002	0.0002	0.0002	0.0002	0.0004	0.0004
0	0.0020	0.0010	0.0031	0.0040	0.0058	0.0040
0	0.0031	0.0040	0.0020	0.0010	0.0040	0.0058
0	0.0001	0.0001	0.0000	0.0001	0.0002	0.0001
0	0.0001	0.0001	0.0001	0.0000	0.0001	0.0002
0	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002
0	0.0001	0.0000	0.0001	0.0001	0.0002	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.8535	0.8535	0.8376	0.8376	0.7851	0.7851	0.7851
0.0320	0.0321	0.0207	0.0501	0.0586	0.0533	0.0343
0.0321	0.0320	0.0333	0.0376	0.0399	0.0586	0.0533
0.0320	0.0321	0.0501	0.0207	0.0343	0.0399	0.0586
0.0321	0.0320	0.0376	0.0333	0.0533	0.0343	0.0399
0.0043	0.0043	0.0040	0.0058	0.0088	0.0054	0.0046
0.0043	0.0043	0.0058	0.0040	0.0046	0.0079	0.0088
0.0004	0.0003	0.0004	0.0004	0.0006	0.0006	0.0006
0.0003	0.0004	0.0004	0.0004	0.0006	0.0006	0.0006
0.0043	0.0043	0.0024	0.0072	0.0079	0.0088	0.0054
0.0043	0.0043	0.0072	0.0024	0.0054	0.0046	0.0079
0.0001	0.0001	0.0002	0.0001	0.0001	0.0002	0.0003
0.0001	0.0001	0.0002	0.0001	0.0002	0.0001	0.0002
0.0001	0.0001	0.0001	0.0002	0.0003	0.0002	0.0001
0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0002
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.7851	0.7256
0.0399	0.0586
0.0343	0.0586
0.0533	0.0586
0.0586	0.0586
0.0079	0.0092
0.0054	0.0092
0.0006	0.0011
0.0006	0.0011
0.0046	0.0092
0.0088	0.0092
0.0002	0.0003
0.0003	0.0003
0.0002	0.0003
0.0001	0.0003
0.0000	0.0000

16x16

6th Power of Transition matrix P

States 0 through 6

1.0000	0.9444	0.9444	0.9444	0.9444	0.8978	0.8978
0	0.0078	0.0111	0.0166	0.0133	0.0182	0.0266
0	0.0133	0.0078	0.0111	0.0166	0.0280	0.0167
0	0.0166	0.0133	0.0078	0.0111	0.0266	0.0182
0	0.0111	0.0166	0.0133	0.0078	0.0167	0.0280
0	0.0011	0.0022	0.0021	0.0010	0.0018	0.0042
0	0.0021	0.0010	0.0011	0.0022	0.0042	0.0018
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0010	0.0011	0.0022	0.0021	0.0028	0.0032
0	0.0022	0.0021	0.0010	0.0011	0.0032	0.0028
0	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000
0	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001
0	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9079	0.9079	0.8978	0.8978	0.8643	0.8643	0.8643
0.0202	0.0202	0.0167	0.0280	0.0362	0.0297	0.0229
0.0202	0.0202	0.0182	0.0266	0.0296	0.0362	0.0297
0.0202	0.0202	0.0280	0.0167	0.0229	0.0296	0.0362
0.0202	0.0202	0.0266	0.0182	0.0297	0.0229	0.0296
0.0027	0.0027	0.0032	0.0028	0.0046	0.0030	0.0035
0.0027	0.0027	0.0028	0.0032	0.0035	0.0050	0.0046
0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003
0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003
0.0027	0.0027	0.0018	0.0042	0.0050	0.0046	0.0030
0.0027	0.0027	0.0042	0.0018	0.0030	0.0035	0.0050
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.8643	0.8257
0.0296	0.0378
0.0229	0.0378
0.0297	0.0378
0.0362	0.0378
0.0050	0.0053
0.0030	0.0053
0.0003	0.0005
0.0003	0.0005
0.0035	0.0053
0.0046	0.0053
0.0001	0.0002
0.0001	0.0002
0.0001	0.0002
0.0001	0.0002
0.0000	0.0000

16x16

7th Power of Transition matrix P

States 0 through 6

1.0000	0.9651	0.9651	0.9651	0.9651	0.9357	0.9357
0	0.0056	0.0082	0.0097	0.0071	0.0110	0.0171
0	0.0071	0.0056	0.0082	0.0097	0.0155	0.0127
0	0.0097	0.0071	0.0056	0.0082	0.0171	0.0110
0	0.0082	0.0097	0.0071	0.0056	0.0127	0.0155
0	0.0009	0.0014	0.0011	0.0007	0.0014	0.0023
0	0.0011	0.0007	0.0009	0.0014	0.0023	0.0014
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0007	0.0009	0.0014	0.0011	0.0015	0.0022
0	0.0014	0.0011	0.0007	0.0009	0.0022	0.0015
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9421	0.9421	0.9357	0.9357	0.9146	0.9146	0.9146
0.0127	0.0127	0.0127	0.0155	0.0215	0.0172	0.0159
0.0127	0.0127	0.0110	0.0171	0.0202	0.0215	0.0172
0.0127	0.0127	0.0155	0.0127	0.0159	0.0202	0.0215
0.0127	0.0127	0.0171	0.0110	0.0172	0.0159	0.0202
0.0017	0.0017	0.0022	0.0015	0.0025	0.0020	0.0025
0.0017	0.0017	0.0015	0.0022	0.0025	0.0030	0.0025
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
0.0017	0.0017	0.0014	0.0023	0.0030	0.0025	0.0020
0.0017	0.0017	0.0023	0.0014	0.0020	0.0025	0.0030
0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0001
0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9146	0.8901
0.0202	0.0240
0.0159	0.0240
0.0172	0.0240
0.0215	0.0240
0.0030	0.0033
0.0020	0.0033
0.0002	0.0003
0.0002	0.0003
0.0025	0.0033
0.0025	0.0033
0.0000	0.0001
0.0001	0.0001
0.0001	0.0001
0.0001	0.0001
0.0000	0.0000

16x16

8th Power of Transition matrix P

States 0 through 6

1.0000	0.9781	0.9781	0.9781	0.9781	0.9596	0.9596
0	0.0041	0.0055	0.0056	0.0041	0.0073	0.0104
0	0.0041	0.0041	0.0055	0.0056	0.0087	0.0090
0	0.0056	0.0041	0.0041	0.0055	0.0104	0.0073
0	0.0055	0.0056	0.0041	0.0041	0.0090	0.0087
0	0.0007	0.0008	0.0006	0.0005	0.0011	0.0013
0	0.0006	0.0005	0.0007	0.0008	0.0013	0.0011
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0005	0.0007	0.0008	0.0006	0.0009	0.0014
0	0.0008	0.0006	0.0005	0.0007	0.0014	0.0009
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9636	0.9636	0.9596	0.9596	0.9463	0.9463	0.9463
0.0080	0.0080	0.0090	0.0087	0.0126	0.0105	0.0109
0.0080	0.0080	0.0073	0.0104	0.0130	0.0126	0.0105
0.0080	0.0080	0.0087	0.0090	0.0109	0.0130	0.0126
0.0080	0.0080	0.0104	0.0073	0.0105	0.0109	0.0130
0.0011	0.0011	0.0014	0.0009	0.0014	0.0013	0.0017
0.0011	0.0011	0.0009	0.0014	0.0017	0.0018	0.0014
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0011	0.0011	0.0011	0.0013	0.0018	0.0014	0.0013
0.0011	0.0011	0.0013	0.0011	0.0013	0.0017	0.0018
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9463	0.9308
0.0130	0.0151
0.0109	0.0151
0.0105	0.0151
0.0126	0.0151
0.0018	0.0020
0.0013	0.0020
0.0001	0.0002
0.0001	0.0002
0.0017	0.0020
0.0014	0.0020
0.0000	0.0001
0.0000	0.0001
0.0000	0.0001
0.0000	0.0001
0.0000	0.0000

16x16

9th Power of Transition matrix P

States 0 through 6

1.0000	0.9862	0.9862	0.9862	0.9862	0.9746	0.9746
0	0.0029	0.0035	0.0032	0.0026	0.0049	0.0062
0	0.0026	0.0029	0.0035	0.0032	0.0052	0.0060
0	0.0032	0.0026	0.0029	0.0035	0.0062	0.0049
0	0.0035	0.0032	0.0026	0.0029	0.0060	0.0052
0	0.0005	0.0005	0.0003	0.0003	0.0007	0.0007
0	0.0003	0.0003	0.0005	0.0005	0.0007	0.0007
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0003	0.0005	0.0005	0.0003	0.0006	0.0009
0	0.0005	0.0003	0.0003	0.0005	0.0009	0.0006
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9772	0.9772	0.9746	0.9746	0.9663	0.9663	0.9663
0.0050	0.0050	0.0060	0.0052	0.0075	0.0067	0.0073
0.0050	0.0050	0.0049	0.0062	0.0081	0.0075	0.0067
0.0050	0.0050	0.0052	0.0060	0.0073	0.0081	0.0075
0.0050	0.0050	0.0062	0.0049	0.0067	0.0073	0.0081
0.0007	0.0007	0.0009	0.0006	0.0009	0.0009	0.0011
0.0007	0.0007	0.0006	0.0009	0.0011	0.0010	0.0009
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0007	0.0007	0.0007	0.0007	0.0010	0.0009	0.0009
0.0007	0.0007	0.0007	0.0007	0.0009	0.0011	0.0010
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9663	0.9565
0.0081	0.0095
0.0073	0.0095
0.0067	0.0095
0.0075	0.0095
0.0010	0.0013
0.0009	0.0013
0.0001	0.0001
0.0001	0.0001
0.0011	0.0013
0.0009	0.0013
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

10th Power of Transition matrix P

States 0 through 6

1.0000	0.9914	0.9914	0.9914	0.9914	0.9841	0.9841
0	0.0019	0.0021	0.0019	0.0017	0.0033	0.0037
0	0.0017	0.0019	0.0021	0.0019	0.0032	0.0038
0	0.0019	0.0017	0.0019	0.0021	0.0037	0.0033
0	0.0021	0.0019	0.0017	0.0019	0.0038	0.0032
0	0.0003	0.0003	0.0002	0.0002	0.0005	0.0004
0	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0002	0.0003	0.0003	0.0002	0.0004	0.0005
0	0.0003	0.0002	0.0002	0.0003	0.0005	0.0004
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9857	0.9857	0.9841	0.9841	0.9788	0.9788	0.9788
0.0031	0.0031	0.0038	0.0032	0.0045	0.0043	0.0048
0.0031	0.0031	0.0033	0.0037	0.0050	0.0045	0.0043
0.0031	0.0031	0.0032	0.0038	0.0048	0.0050	0.0045
0.0031	0.0031	0.0037	0.0033	0.0043	0.0048	0.0050
0.0004	0.0004	0.0005	0.0004	0.0006	0.0006	0.0007
0.0004	0.0004	0.0004	0.0005	0.0007	0.0006	0.0006
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0004	0.0004	0.0005	0.0004	0.0006	0.0006	0.0006
0.0004	0.0004	0.0004	0.0005	0.0006	0.0007	0.0006
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9788	0.9727
0.0050	0.0060
0.0048	0.0060
0.0043	0.0060
0.0045	0.0060
0.0006	0.0008
0.0006	0.0008
0.0000	0.0001
0.0000	0.0001
0.0007	0.0008
0.0006	0.0008
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

11th Power of Transition matrix P

States 0 through 6

1.0000	0.9946	0.9946	0.9946	0.9946	0.9900	0.9900
0	0.0013	0.0013	0.0011	0.0011	0.0022	0.0022
0	0.0011	0.0013	0.0013	0.0011	0.0020	0.0024
0	0.0011	0.0011	0.0013	0.0013	0.0022	0.0022
0	0.0013	0.0011	0.0011	0.0013	0.0024	0.0020
0	0.0002	0.0002	0.0001	0.0002	0.0003	0.0003
0	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0002	0.0002	0.0002	0.0001	0.0003	0.0003
0	0.0002	0.0001	0.0002	0.0002	0.0003	0.0003
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9910	0.9910	0.9900	0.9900	0.9867	0.9867	0.9867
0.0020	0.0020	0.0024	0.0020	0.0028	0.0028	0.0030
0.0020	0.0020	0.0022	0.0022	0.0030	0.0028	0.0028
0.0020	0.0020	0.0020	0.0024	0.0030	0.0030	0.0028
0.0020	0.0020	0.0022	0.0022	0.0028	0.0030	0.0030
0.0003	0.0003	0.0003	0.0003	0.0004	0.0004	0.0004
0.0003	0.0003	0.0003	0.0003	0.0004	0.0004	0.0004
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0003	0.0003	0.0003	0.0003	0.0004	0.0004	0.0004
0.0003	0.0003	0.0003	0.0003	0.0004	0.0004	0.0004
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9867	0.9829
0.0030	0.0038
0.0030	0.0038
0.0028	0.0038
0.0028	0.0038
0.0004	0.0005
0.0004	0.0005
0.0000	0.0000
0.0000	0.0000
0.0004	0.0005
0.0004	0.0005
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

12th Power of Transition matrix P

States 0 through 6

1.0000	0.9966	0.9966	0.9966	0.9966	0.9937	0.9937
0	0.0008	0.0008	0.0007	0.0007	0.0014	0.0013
0	0.0007	0.0008	0.0008	0.0007	0.0013	0.0014
0	0.0007	0.0007	0.0008	0.0008	0.0013	0.0014
0	0.0008	0.0007	0.0007	0.0008	0.0014	0.0013
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9943	0.9943	0.9937	0.9937	0.9917	0.9917	0.9917
0.0012	0.0012	0.0014	0.0013	0.0018	0.0018	0.0019
0.0012	0.0012	0.0014	0.0013	0.0018	0.0018	0.0018
0.0012	0.0012	0.0013	0.0014	0.0019	0.0018	0.0018
0.0012	0.0012	0.0013	0.0014	0.0018	0.0019	0.0018
0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0002
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003
0.0002	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9917	0.9892
0.0018	0.0024
0.0019	0.0024
0.0018	0.0024
0.0018	0.0024
0.0002	0.0003
0.0003	0.0003
0.0000	0.0000
0.0000	0.0000
0.0002	0.0003
0.0002	0.0003
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

13th Power of Transition matrix P

States 0 through 6

1.0000	0.9979	0.9979	0.9979	0.9979	0.9961	0.9961
0	0.0005	0.0005	0.0004	0.0005	0.0009	0.0008
0	0.0005	0.0005	0.0005	0.0004	0.0008	0.0009
0	0.0004	0.0005	0.0005	0.0005	0.0008	0.0009
0	0.0005	0.0004	0.0005	0.0005	0.0009	0.0008
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9964	0.9964	0.9961	0.9961	0.9948	0.9948	0.9948
0.0008	0.0008	0.0009	0.0008	0.0011	0.0012	0.0012
0.0008	0.0008	0.0009	0.0008	0.0011	0.0011	0.0012
0.0008	0.0008	0.0008	0.0009	0.0012	0.0011	0.0011
0.0008	0.0008	0.0008	0.0009	0.0012	0.0012	0.0011
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0002
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9948	0.9932
0.0011	0.0015
0.0012	0.0015
0.0012	0.0015
0.0011	0.0015
0.0001	0.0002
0.0002	0.0002
0.0000	0.0000
0.0000	0.0000
0.0002	0.0002
0.0002	0.0002
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

14th Power of Transition matrix P

States 0 through 6

1.0000	0.9987	0.9987	0.9987	0.9987	0.9975	0.9975
0	0.0003	0.0003	0.0003	0.0003	0.0006	0.0005
0	0.0003	0.0003	0.0003	0.0003	0.0005	0.0005
0	0.0003	0.0003	0.0003	0.0003	0.0005	0.0006
0	0.0003	0.0003	0.0003	0.0003	0.0005	0.0005
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9978	0.9978	0.9975	0.9975	0.9967	0.9967	0.9967
0.0005	0.0005	0.0005	0.0005	0.0007	0.0007	0.0007
0.0005	0.0005	0.0006	0.0005	0.0007	0.0007	0.0007
0.0005	0.0005	0.0005	0.0005	0.0007	0.0007	0.0007
0.0005	0.0005	0.0005	0.0006	0.0007	0.0007	0.0007
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9967	0.9958
0.0007	0.0009
0.0007	0.0009
0.0007	0.0009
0.0007	0.0009
0.0001	0.0001
0.0001	0.0001
0.0000	0.0000
0.0000	0.0000
0.0001	0.0001
0.0001	0.0001
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

15th Power of Transition matrix P

States 0 through 6

1.0000	0.9992	0.9992	0.9992	0.9992	0.9984	0.9984
0	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003
0	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003
0	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003
0	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9986	0.9986	0.9984	0.9984	0.9979	0.9979	0.9979
0.0003	0.0003	0.0003	0.0003	0.0005	0.0005	0.0005
0.0003	0.0003	0.0003	0.0003	0.0004	0.0005	0.0005
0.0003	0.0003	0.0003	0.0003	0.0005	0.0004	0.0005
0.0003	0.0003	0.0003	0.0003	0.0005	0.0005	0.0004
0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9979	0.9973
0.0004	0.0006
0.0005	0.0006
0.0005	0.0006
0.0005	0.0006
0.0001	0.0001
0.0001	0.0001
0.0000	0.0000
0.0000	0.0000
0.0001	0.0001
0.0001	0.0001
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

16th Power of Transition matrix P

States 0 through 6

1.0000	0.9995	0.9995	0.9995	0.9995	0.9990	0.9990
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9991	0.9991	0.9990	0.9990	0.9987	0.9987	0.9987
0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003
0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003
0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003
0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9987	0.9983
0.0003	0.0004
0.0003	0.0004
0.0003	0.0004
0.0003	0.0004
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

17th Power of Transition matrix P

States 0 through 6

1.0000	0.9997	0.9997	0.9997	0.9997	0.9994	0.9994
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9994	0.9994	0.9994	0.9994	0.9992	0.9992	0.9992
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.000C	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9992	0.9989
0.0002	0.0002
0.0002	0.0002
0.0002	0.0002
0.0002	0.0002
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

18th Power of Transition matrix P

States 0 through 6

1.0000	0.9998	0.9998	0.9998	0.9998	0.9996	0.9996
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0.9995
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9995	0.9993
0.0001	0.0001
0.0001	0.0001
0.0001	0.0001
0.0001	0.0001
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

19th Power of Transition matrix P

States 0 through 6

1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997
0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9997	0.9996
0.0001	0.0001
0.0001	0.0001
0.0001	0.0001
0.0001	0.0001
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

20th Power of Transition matrix P

States 0 through 6

1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9998
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9998	0.9997
0.0000	0.0001
0.0000	0.0001
0.0000	0.0001
0.0000	0.0001
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

21st Power of Transition matrix P

States 0 through 6

1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9999	0.9998
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

22nd Power of Transition matrix P

States 0 through 6

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

0.9999	0.9999
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

23rd Power of Transition matrix P

States 0 through 6

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

1.0000	0.9999
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

24th Power of Transition matrix P

States 0 through 6

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 7 through 13

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

States 14 through 15

1.0000	1.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000

16x16

Note : The (3/4 nodes bad & 2/10 spares bad) system is repaired after 24 applications of the Algorithm_1.

AGGREGATED TRANSITION MATRIX OF THE 4 NODES SYSTEM

$$Pa=\begin{bmatrix} 1 & 0.4087 & 0.1302 & 0.0410 & 0.0171 \\ 0 & 0.5478 & 0.5934 & 0.3374 & 0.1548 \\ 0 & 0.0435 & 0.2654 & 0.5006 & 0.3906 \\ 0 & 0 & 0.0110 & 0.1190 & 0.3500 \\ 0 & 0 & 0 & 0.0020 & 0.0875 \end{bmatrix}_{5x5}$$

5 EIGENVECTORS OF THE AGGREGATED TRANSITION MATRIX

$$V0=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{5X1} \quad V1=\begin{bmatrix} 1.0000 \\ -0.8864 \\ -0.1112 \\ -0.0024 \\ -0.0000 \end{bmatrix}_{5X1} V2=\begin{bmatrix} -0.4434 \\ 1.0000 \\ -0.5042 \\ -0.0517 \\ -0.0007 \end{bmatrix}_{5X1} V3=\begin{bmatrix} -0.3413 \\ 1.0000 \\ -0.8485 \\ 0.1700 \\ 0.0197 \end{bmatrix}_{5X1} V4=\begin{bmatrix} 0.3062 \\ -0.9890 \\ 1.0000 \\ -0.3429 \\ 0.0256 \end{bmatrix}_{5X1}$$

5 EIGENVALUES OF	E0=1.0000
THE AGGREGATED	E1=0.6232
TRANSITION MATRIX	E2=0.2311
	E3=0.1047
	E4=0.0607

INVERSE MATRIX OF EIGENVECTORS

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & -0.9218 & -1.5970 & -2.2140 & -2.8774 \\ 0 & 0.1574 & -1.1459 & -4.9117 & -14.9232 \\ 0 & 0.0168 & -0.1715 & 1.5965 & 28.6913 \\ 0 & -0.0089 & 0.0992 & -1.3679 & 16.5004 \end{bmatrix}_{5x5}$$

1st Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.4087 & 0.1302 & 0.0410 & 0.0171 \\ 0 & 0.5478 & 0.5934 & 0.3374 & 0.1548 \\ 0 & 0.0435 & 0.2654 & 0.5006 & 0.3906 \\ 0 & 0 & 0.0110 & 0.1190 & 0.3500 \\ 0 & 0 & 0 & 0.0020 & 0.0875 \end{bmatrix}_{5x5}$$

2nd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.6382 & 0.4077 & 0.2490 & 0.1471 \\ 0 & 0.3259 & 0.4863 & 0.5223 & 0.4482 \\ 0 & 0.0354 & 0.1018 & 0.2079 & 0.3198 \\ 0 & 0.0005 & 0.0042 & 0.0204 & 0.0766 \\ 0 & 0 & 0.0000 & 0.0004 & 0.0084 \end{bmatrix}_{5x5}$$

3rd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.7761 & 0.6199 & 0.4904 & 0.3752 \\ 0 & 0.1997 & 0.3282 & 0.4164 & 0.4624 \\ 0 & 0.0238 & 0.0503 & 0.0883 & 0.1460 \\ 0 & 0.0004 & 0.0016 & 0.0049 & 0.0156 \\ 0 & 0.0000 & 0.0000 & 0.0001 & 0.0009 \end{bmatrix}_{5x5}$$

4th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.8608 & 0.7606 & 0.6723 & 0.5838 \\ 0 & 0.1237 & 0.2102 & 0.2821 & 0.3453 \\ 0 & 0.0152 & 0.0284 & 0.0440 & 0.0670 \\ 0 & 0.0003 & 0.0008 & 0.0016 & 0.0038 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \end{bmatrix}_{5x5}$$

5th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9133 & 0.8503 & 0.7934 & 0.7338 \\ 0 & 0.0769 & 0.1323 & 0.1812 & 0.2302 \\ 0 & 0.0096 & 0.0171 & 0.0247 & 0.0347 \\ 0 & 0.0002 & 0.0004 & 0.0007 & 0.0012 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

6th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9460 & 0.9066 & 0.8707 & 0.8325 \\ 0 & 0.0479 & 0.0827 & 0.1142 & 0.1471 \\ 0 & 0.0060 & 0.0105 & 0.0148 & 0.0198 \\ 0 & 0.0001 & 0.0002 & 0.0004 & 0.0005 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

7th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9664 & 0.9417 & 0.9193 & 0.8952 \\ 0 & 0.0298 & 0.0516 & 0.0714 & 0.0926 \\ 0 & 0.0037 & 0.0065 & 0.0091 & 0.0119 \\ 0 & 0.0001 & 0.0001 & 0.0002 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

8th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9790 & 0.9637 & 0.9497 & 0.9346 \\ 0 & 0.0186 & 0.0322 & 0.0446 & 0.0579 \\ 0 & 0.0023 & 0.0040 & 0.0056 & 0.0073 \\ 0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

9th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9869 & 0.9774 & 0.9686 & 0.9592 \\ 0 & 0.0116 & 0.0201 & 0.0278 & 0.0361 \\ 0 & 0.0015 & 0.0025 & 0.0035 & 0.0045 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

10th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9919 & 0.9859 & 0.9804 & 0.9746 \\ 0 & 0.0072 & 0.0125 & 0.0173 & 0.0225 \\ 0 & 0.0009 & 0.0016 & 0.0022 & 0.0028 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

11th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9949 & 0.9912 & 0.9878 & 0.9842 \\ 0 & 0.0045 & 0.0078 & 0.0108 & 0.0140 \\ 0 & 0.0006 & 0.0010 & 0.0014 & 0.0018 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

12th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9968 & 0.9945 & 0.9924 & 0.9901 \\ 0 & 0.0028 & 0.0049 & 0.0067 & 0.0087 \\ 0 & 0.0004 & 0.0006 & 0.0008 & 0.0011 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

13th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9980 & 0.9966 & 0.9953 & 0.9939 \\ 0 & 0.0017 & 0.0030 & 0.0042 & 0.0055 \\ 0 & 0.0002 & 0.0004 & 0.0005 & 0.0007 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

14th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9988 & 0.9979 & 0.9971 & 0.9962 \\ 0 & 0.0011 & 0.0019 & 0.0026 & 0.0034 \\ 0 & 0.0001 & 0.0002 & 0.0003 & 0.0004 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

15th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9992 & 0.9987 & 0.9982 & 0.9976 \\ 0 & 0.0007 & 0.0012 & 0.0016 & 0.0021 \\ 0 & 0.0001 & 0.0001 & 0.0002 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

16th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9995 & 0.9992 & 0.9989 & 0.9985 \\ 0 & 0.0004 & 0.0007 & 0.0010 & 0.0013 \\ 0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

17th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9997 & 0.9995 & 0.9993 & 0.9991 \\ 0 & 0.0003 & 0.0005 & 0.0006 & 0.0008 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

18th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9998 & 0.9997 & 0.9996 & 0.9994 \\ 0 & 0.0002 & 0.0003 & 0.0004 & 0.0005 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

19th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9998 & 0.9997 & 0.9996 \\ 0 & 0.0001 & 0.0002 & 0.0002 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

20th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9999 & 0.9998 & 0.9998 \\ 0 & 0.0001 & 0.0001 & 0.0002 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

21st Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 0.9999 & 0.9999 & 0.9999 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

22nd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 0.9999 & 0.9999 \\ 0 & 0.0000 & 0.0000 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

23rd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 & 0.9999 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

24th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{5x5}$$

Note : After aggregation the (3/4 nodes bad & 2/10 spares bad) system is also repaired after 24 applications of the Algorithm_1.

APPENDIX D

4/5 NODES BAD & 2/10 SPARES BAD SYSTEM CASE RESULTS

This appendix contains matrix manipulations for five node system. (Faulty nodes=4/Faulty spares=2/Fault-free spares=8)

$$
Pa = \begin{bmatrix}
1 & 0.3214 & 0.0804 & 0.0236 & 0.0091 & 0.0052 \\
0 & 0.6072 & 0.5337 & 0.2843 & 0.1354 & 0.0687 \\
0 & 0.0714 & 0.3578 & 0.5235 & 0.4200 & 0.2504 \\
0 & 0 & 0.0281 & 0.1629 & 0.3681 & 0.3841 \\
0 & 0 & 0 & 0.0057 & 0.0664 & 0.2448 \\
0 & 0 & 0 & 0 & 0.0010 & 0.0468
\end{bmatrix}_{6x6}
$$

6 EIGENVECTORS OF THE AGGREGATED TRANSITION MATRIX

$$
\begin{array}{cccccc}
V0 & V1 & V2 & V3 & V4 & V5 \\
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} &
\begin{bmatrix} 1.0000 \\ -0.8185 \\ -0.1727 \\ -0.0087 \\ -0.0001 \\ -0.0000 \end{bmatrix} &
\begin{bmatrix} -0.4088 \\ 1.0000 \\ -0.4934 \\ -0.0956 \\ -0.0022 \\ -0.0000 \end{bmatrix} &
\begin{bmatrix} 0.2450 \\ -0.8998 \\ 1.0000 \\ -0.3074 \\ -0.0373 \\ -0.0005 \end{bmatrix} &
\begin{bmatrix} 0.1412 \\ -0.6314 \\ 1.0000 \\ -0.6788 \\ 0.1797 \\ -0.0108 \end{bmatrix} &
\begin{bmatrix} 0.1673 \\ -0.7042 \\ 1.0000 \\ -0.5468 \\ 0.0745 \\ 0.0092 \end{bmatrix}
\end{array}
$$

6 EIGENVALUES OF	E0=1.0000
THE AGGREGATED	E1=0.7228
TRANSITION MATRIX	E2=0.3164
	E3=0.1168
	E4=0.0302
	E5=0.0549

INVERSE MATRIX OF EIGENVECTORS

$$
\begin{bmatrix}
1 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\
0 & -0.8959 & -1.4509 & -1.8332 & -2.1448 & -2.4467 \\
0 & 0.2321 & -0.9455 & -3.0155 & -5.9449 & -10.5136 \\
0 & -0.0330 & 0.2269 & -1.3184 & -8.5405 & -36.3020 \\
0 & -0.0025 & 0.0201 & -0.1876 & 2.6422 & -34.8325 \\
0 & -0.0046 & 0.0358 & -0.2984 & 2.5945 & 65.5244
\end{bmatrix}_{6X6}
$$

1st Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.3214 & 0.0804 & 0.0236 & 0.0091 & 0.0052 \\ 0 & 0.6072 & 0.5337 & 0.2843 & 0.1354 & 0.0687 \\ 0 & 0.0714 & 0.3578 & 0.5235 & 0.4200 & 0.2504 \\ 0 & 0 & 0.0281 & 0.1629 & 0.3681 & 0.3841 \\ 0 & 0 & 0 & 0.0057 & 0.0664 & 0.2448 \\ 0 & 0 & 0 & 0 & 0.0010 & 0.0468 \end{bmatrix}_{6X6}$$

2nd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.5223 & 0.2814 & 0.1610 & 0.0957 & 0.0589 \\ 0 & 0.4068 & 0.5230 & 0.4991 & 0.4201 & 0.3209 \\ 0 & 0.0689 & 0.1808 & 0.2953 & 0.3808 & 0.4101 \\ 0 & 0.0020 & 0.0146 & 0.0433 & 0.0966 & 0.1777 \\ 0 & 0 & 0.0002 & 0.0013 & 0.0068 & 0.0299 \\ 0 & 0 & 0 & 0.0000 & 0.0001 & 0.0024 \end{bmatrix}_{6X6}$$

3rd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.6586 & 0.4643 & 0.3461 & 0.2637 & 0.1995 \\ 0 & 0.2844 & 0.4183 & 0.4731 & 0.4867 & 0.4685 \\ 0 & 0.0547 & 0.1098 & 0.1645 & 0.2197 & 0.2758 \\ 0 & 0.0023 & 0.0075 & 0.0158 & 0.0290 & 0.0524 \\ 0 & 0.0000 & 0.0001 & 0.0003 & 0.0010 & 0.0036 \\ 0 & 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \end{bmatrix}_{6X6}$$

4th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.7545 & 0.6078 & 0.5118 & 0.4384 & 0.3736 \\ 0 & 0.2025 & 0.3147 & 0.3797 & 0.4211 & 0.4471 \\ 0 & 0.0411 & 0.0731 & 0.1011 & 0.1289 & 0.1611 \\ 0 & 0.0019 & 0.0043 & 0.0073 & 0.0113 & 0.0177 \\ 0 & 0.0000 & 0.0000 & 0.0001 & 0.0002 & 0.0006 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

5th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.8229 & 0.7149 & 0.6421 & 0.5844 & 0.5306 \\ 0 & 0.1454 & 0.2314 & 0.2866 & 0.3278 & 0.3626 \\ 0 & 0.0302 & 0.0509 & 0.0672 & 0.0822 & 0.0991 \\ 0 & 0.0015 & 0.0028 & 0.0041 & 0.0055 & 0.0076 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

6th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.8721 & 0.7934 & 0.7397 & 0.6965 & 0.6553 \\ 0 & 0.1048 & 0.1685 & 0.2110 & 0.2445 & 0.2752 \\ 0 & 0.0220 & 0.0362 & 0.0466 & 0.0558 & 0.0654 \\ 0 & 0.0011 & 0.0019 & 0.0026 & 0.0032 & 0.0041 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

7th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9076 & 0.8505 & 0.8114 & 0.7796 & 0.7491 \\ 0 & 0.0757 & 0.1222 & 0.1538 & 0.1791 & 0.2032 \\ 0 & 0.0159 & 0.0260 & 0.0331 & 0.0391 & 0.0452 \\ 0 & 0.0008 & 0.0013 & 0.0017 & 0.0021 & 0.0025 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

8th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9332 & 0.8919 & 0.8635 & 0.8404 & 0.8181 \\ 0 & 0.0547 & 0.0884 & 0.1115 & 0.1302 & 0.1482 \\ 0 & 0.0115 & 0.0187 & 0.0237 & 0.0279 & 0.0320 \\ 0 & 0.0006 & 0.0010 & 0.0012 & 0.0015 & 0.0017 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

9th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9517 & 0.9218 & 0.9013 & 0.8845 & 0.8683 \\ 0 & 0.0395 & 0.0639 & 0.0807 & 0.0944 & 0.1076 \\ 0 & 0.0083 & 0.0135 & 0.0171 & 0.0200 & 0.0229 \\ 0 & 0.0004 & 0.0007 & 0.0009 & 0.0010 & 0.0012 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

10th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9651 & 0.9435 & 0.9286 & 0.9165 & 0.9048 \\ 0 & 0.0286 & 0.0462 & 0.0584 & 0.0683 & 0.0779 \\ 0 & 0.0060 & 0.0098 & 0.0123 & 0.0145 & 0.0165 \\ 0 & 0.0003 & 0.0005 & 0.0006 & 0.0007 & 0.0008 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6x6}$$

11th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9748 & 0.9592 & 0.9484 & 0.9396 & 0.9311 \\ 0 & 0.0206 & 0.0334 & 0.0422 & 0.0494 & 0.0563 \\ 0 & 0.0044 & 0.0071 & 0.0089 & 0.0104 & 0.0119 \\ 0 & 0.0002 & 0.0004 & 0.0005 & 0.0005 & 0.0006 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

12th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9818 & 0.9705 & 0.9627 & 0.9564 & 0.9502 \\ 0 & 0.0149 & 0.0242 & 0.0305 & 0.0357 & 0.0407 \\ 0 & 0.0031 & 0.0051 & 0.0064 & 0.0075 & 0.0086 \\ 0 & 0.0002 & 0.0003 & 0.0003 & 0.0004 & 0.0004 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

13th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9868 & 0.9787 & 0.9730 & 0.9685 & 0.9640 \\ 0 & 0.0108 & 0.0175 & 0.0221 & 0.0258 & 0.0295 \\ 0 & 0.0023 & 0.0037 & 0.0047 & 0.0054 & 0.0062 \\ 0 & 0.0001 & 0.0002 & 0.0002 & 0.0003 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

14th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9905 & 0.9846 & 0.9805 & 0.9772 & 0.9740 \\ 0 & 0.0078 & 0.0126 & 0.0160 & 0.0187 & 0.0213 \\ 0 & 0.0016 & 0.0027 & 0.0034 & 0.0039 & 0.0045 \\ 0 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

15th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9931 & 0.9889 & 0.9859 & 0.9835 & 0.9812 \\ 0 & 0.0056 & 0.0091 & 0.0115 & 0.0135 & 0.0154 \\ 0 & 0.0012 & 0.0019 & 0.0024 & 0.0028 & 0.0032 \\ 0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

16th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9950 & 0.9919 & 0.9898 & 0.9881 & 0.9864 \\ 0 & 0.0041 & 0.0066 & 0.0083 & 0.0098 & 0.0111 \\ 0 & 0.0009 & 0.0014 & 0.0018 & 0.0021 & 0.0023 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

17th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9964 & 0.9942 & 0.9926 & 0.9914 & 0.9902 \\ 0 & 0.0029 & 0.0048 & 0.0060 & 0.0070 & 0.0080 \\ 0 & 0.0006 & 0.0010 & 0.0013 & 0.0015 & 0.0017 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

18th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9974 & 0.9958 & 0.9947 & 0.9938 & 0.9929 \\ 0 & 0.0021 & 0.0034 & 0.0044 & 0.0051 & 0.0058 \\ 0 & 0.0004 & 0.0007 & 0.0009 & 0.0011 & 0.0012 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

19th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9981 & 0.9970 & 0.9962 & 0.9955 & 0.9949 \\ 0 & 0.0015 & 0.0025 & 0.0031 & 0.0037 & 0.0042 \\ 0 & 0.0003 & 0.0005 & 0.0007 & 0.0008 & 0.0009 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

20th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9986 & 0.9978 & 0.9972 & 0.9967 & 0.9963 \\ 0 & 0.0011 & 0.0018 & 0.0023 & 0.0027 & 0.0030 \\ 0 & 0.0002 & 0.0004 & 0.0005 & 0.0006 & 0.0006 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

21st Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9990 & 0.9984 & 0.9980 & 0.9976 & 0.9973 \\ 0 & 0.0008 & 0.0013 & 0.0016 & 0.0019 & 0.0022 \\ 0 & 0.0002 & 0.0003 & 0.0003 & 0.0004 & 0.0005 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

22nd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9993 & 0.9989 & 0.9985 & 0.9983 & 0.9981 \\ 0 & 0.0006 & 0.0009 & 0.0012 & 0.0014 & 0.0016 \\ 0 & 0.0001 & 0.0002 & 0.0003 & 0.0003 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

23rd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9995 & 0.9992 & 0.9990 & 0.9988 & 0.9986 \\ 0 & 0.0004 & 0.0007 & 0.0009 & 0.0010 & 0.0011 \\ 0 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

24th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9996 & 0.9994 & 0.9992 & 0.9991 & 0.9990 \\ 0 & 0.0003 & 0.0005 & 0.0006 & 0.0007 & 0.0008 \\ 0 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

25th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9997 & 0.9996 & 0.9995 & 0.9994 & 0.9993 \\ 0 & 0.0002 & 0.0004 & 0.0004 & 0.0005 & 0.0006 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

26th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9998 & 0.9997 & 0.9996 & 0.9995 & 0.9995 \\ 0 & 0.0002 & 0.0003 & 0.0003 & 0.0004 & 0.0004 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

27th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9998 & 0.9997 & 0.9997 & 0.9996 \\ 0 & 0.0001 & 0.0002 & 0.0002 & 0.0003 & 0.0003 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

28th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9998 & 0.9998 & 0.9998 & 0.9997 \\ 0 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

29th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9999 & 0.9999 & 0.9998 & 0.9998 \\ 0 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0002 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

30th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

31st Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 0.9999 & 0.9999 & 0.9999 & 0.9999 \\ 0 & 0.0000 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

32nd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 0.9999 & 0.9999 & 0.9999 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0001 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

33rd Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.9999 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

34th Power of the aggregated transition matrix Pa

$$\begin{bmatrix} 1 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}_{6X6}$$

Note : After aggregation the (4/5 nodes bad & 2/10 spares bad) system is also repaired after 34 applications of the Algorithm_1.

LIST OF REFERENCES

1. F.P. Preparata, G. Metze, and R.T. Chien, "On the connection assignment problem of diagnosable systems." IEEE Trans. Comput. Vol. C-16. pp. 848-854, Dec. 1967.

2. C.R. Kime, "An analysis model for digital system diagnosis." IEEE Trans. Comput. Vol. C-19. pp. 1063-1073, Nov. 1970.

3. S.L. Hakimi, S.N. Maheshwari, and A.T. Amin, "On a graph model of diagnosable system." Proc. 4th Annual Pittsburgh Conference on Modeling & Simulation, pp. 220-233, Apr. 1973.

4. S.L. Hakimi and A.T. Amin, "Characterization of the connection assignment of diagnosable systems." IEEE Trans. Comput. Vol. C-23. pp. 86-88, Jan. 1974.

5. J.D. Russell and C.R. Kime, "System fault diagnosis: A closure and diagnosability with repair." IEEE Trans. Comput. Vol. C-24, pp. 1078-1088, Nov. 1975.

6. A.D. Friedman, "A new measure of digital system diagnosis." Digest of papers. 1975 Int. Symp. Fault-Tolerant Computing, pp. 167-170, Jun. 1975.

7. J.P. Hayes, "A graph model for fault-tolerant computing system." IEEE Trans. Comput. Vol. C-25 pp. 875-884, Sept. 1976.

8. F. Barsi, F. Grandoni, and P. Maestrini, "A theory of diagnosability of digital systems." IEEE Trans. Comput. Vol. C-25, pp. 585-593, Jun. 1976.

9. S. N. Maheshwari, S. L. Hakimi, "On models for diagnosable systems and probabilistic fault diagnosis." IEEE Trans. Comput. Vol. C-24, pp. 228-236, Jan. 1976.

10. S. Mallela and G.M. Masson, "Diagnosable systems for intermittent faults." IEEE Trans. Comput. Vol. C-27, pp. 560-566, Jun. 1978.

11. C.R. Kime, "An abstract model for digital system fault diagnosis." IEEE Trans. Comput. Vol. C-28, pp. 754-767, Oct. 1979.

12. S. Karunanithi and A. D. Friedman, "Analysis of digital systems using a new measure of systems diagnosis," IEEE Trans. Comput. Vol. C-28, pp. 121-133, Feb. 1979.

13. J.E. Smith, "Universal systems diagnosis algorithms," IEEE Trans. Comput. Vol. C-28, pp. 374-378, May 1979.

14. K.-Y. Chwa and S.L. Hakimi, "On fault identification in diagnosable systems," IEEE Trans. Comput. Vol. C-30, pp. 414-422, Jun. 1981.

15. J.T. Butler, "Speed-efficiency-complexity tradeoffs in universal diagnosis algorithms," IEEE Trans. Comput. Vol. C-30, pp. 590-595, Aug. 1981.

16. J.T. Butler, "Diagnosis of intermittently and permanently faulty processors in a multiprocessing system using three-valued functions," 12th Inter. Symp. on Multiple-Valued Logic, pp. 122-128, May 1982.

17. J.T. Butler, "Relations among system diagnosis models with three valued test outcomes," Proc. of 13th Inter. Sym. on Multi-Valued Logic, pp. 350-355, May 1983.

18. J.T. Butler and J.-K. Lee, "Analysis of the effect of faulty spare processors on the performance of systems diagnosis algorithm," Dept. Electr. Eng. and Comput. Sci. Northwestern University Evanston, Il, Aug. 1986.

19. J.-K. Lee, "A realistic approach to the design of fault tolerant multi-processing computer systems," Ph.D. dissertation, Dept. Electr. Eng. and Comput. Sci. Northwestern University, Evanston, Il, Aug. 1986.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93943-5100	2
3.	Department Chairman 62 Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, California 93943	1
4.	Professor J.T. Butler, Code 62Bu Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, California 93943	5
5.	Professor Mitchell L. Cotton, Code 62Cc Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, California 93943	2
6.	LTJG. Mustafa Paktuna Marmara Cad. 158/6 Koca. M. Pasa Istanbul, Turkey	6
7.	Deniz Harb Okulu Komutanligi Okul Kutuphanesi ve Elektrik Bolumu Kutuphanesi Tuzla, Istanbul, Turkey	2
8.	LT. Ibrahim Dincer Muhabere Okulu Ogretim Kurulu Mamak, Ankara, Turkey	1

9. Dr. Andre von Tibborg 1
Code 1133
ONR
300 N. Quincy
Arlington, VA 22217

10. Dr. George Abraham 1
Code 7500
NRL 4555 Overlook Ave., S.W.
Washington, DC 20375

11. Dr. Lou Schmid 1
ONT 20T
800 N. Quincy Ave., Room 811
Arlington, VA 22217

END
DATE
FILMED
8-88
DTIC